Yu. I. Vitinskii

SOLAR-ACTIVITY FORECASTING

TRANSLATED FROM RUSSIAN

Published for the U.S. National Aeronautics and Space Administration
and the National Science Foundation, Washington, D.C.
by the Israel Program for Scientific Translations

AKADEMIYA NAUK SSSR
Academy of Sciences of the USSR

Yu. I. VITINSKII

SOLAR-ACTIVITY FORECASTING

(Prognozy solnechnoi aktivnosti)

Izdatel'stvo Akademii Nauk SSSR
Leningrad 1962

Translated from Russian

Israel Program for Scientific Translations
Jerusalem 1965

NASA TT F-289
TT 65-50115

Published Pursuant to an Agreement with
THE NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
and
THE NATIONAL SCIENCE FOUNDATION, WASHINGTON, D. C.

IPST Cat. No. 1400

Translated by Z. Lerman
Edited by R. Hardin

Printed in Jerusalem at the Sivan Press
Binding: K. Wiener

Price: $4.00

Available from the
U.S. DEPARTMENT OF COMMERCE
Clearinghouse for Federal Scientific and Technical Information
Springfield, Va. 22151

IX/18/3.5

CONTENTS

INTRODUCTION

The effects which solar activity has on the various layers of the earth's atmosphere, particularly the ionosphere, play an important part in our daily life. Some of these effects, to name only a few, are variations in radio-communication conditions, disastrous interruptions of radio communication, geomagnetic storms and disturbances, changes in climatic conditions, and polar auroras. We will not consider these subjects in detail here, since such a comprehensive study is not the object of this book and since the interested reader can find the relevant information elsewhere. It should be noted, however, that as the sun-earth problem becomes studied more and more the number of these related questions must increase.

A normal economic life for a country is impossible without advance knowledge of the conditions which will prevail in the various atmospheric levels at any given time. However, since these layers are affected by solar activity, the level of this activity must also be calculated in advance. This is particularly important for the prediction of conditions in the ionosphere, which is the part of the atmosphere affected directly by solar activity. Consequently, the development of methods for forecasting solar activity is of primary importance in solving a number of problems related to the national economy.

Obviously, the more basic knowledge we have about any given phenomenon, the simpler it is to predict it. In this respect, however, we find ourselves in a very complicated position. Astrophysical and radioastronomical observations of the sun enable us to study only the solar surface and solar atmosphere. Thus, when describing the internal structure of the sun, we must rely almost exclusively on hypotheses. Moreover, since knowledge concerning the internal solar structure and the processes occurring in the solar interior is not available, it is impossible to explain the mechanism of solar activity. Although some progress has recently been made in this direction, a general theory, based to a large extent on the various hypotheses, has still not been formulated.

Therefore, given the present state of knowledge of solar activity, solar scientists must use empiricostatistical methods of prediction. This in turn means that long series of similar observations of the various solar formations must be available. Until recently, most such observations were classified according to various solar-activity indexes which do not satisfy rigorous physical standards. The two longest of these series are the Zurich series of Wolf numbers, or relative spot numbers (starting with 1749), and the Greenwich series of sunspot-group areas (starting with 1874). In actual practice, these are the only two indexes that can be used to predict solar activity, and so only these indexes will be discussed here. It should be noted, by the way, that comparisons with geophysical indexes have shown that, to a rough approximation, the Wolf numbers mainly characterize the ultraviolet [wave] component of solar radiation, and the spot areas mainly determine the corpuscular component.

Relative sunspot numbers were introduced by Wolf in Zurich in 1849 and are therefore often called Wolf numbers. They are usually defined as

$$W = k(10g + f), \tag{1}$$

where g is the number of sunspot groups and f is the total number of spots in all the groups. The coefficient k is determined from a comparison of different observation series. It depends on the visibility conditions, the apparatus used, and the method of observation, as well as on such subjective factors as observer fatigue, the way in which the sunspots are combined into groups, and the nucleus count.

All these features of Wolf numbers indicate that this solar-activity index is a fairly subjective one. However, while taking this into account, the Zurich researchers have made every effort to conserve the Zurich system of relative sunspot numbers. To this end long-term simultaneous sunspot observations have been organized by the chief observer and his successor and the results of these observations compared. It should be mentioned that in Wolfer's time the value of the coefficient k was changed. Instead of Wolf's value of $k=1$, Wolfer gave $k=0.6$. The coefficient was subsequently kept constant, the above change being due to an alteration in the method for counting sunspot nuclei.

It may be questioned whether the Zurich system, with all its precautions, ensures homogeneity of the Wolf numbers. Various doubts as to this have recently been expressed. The problem is that, as Slonim has pointed out, the coefficient k can change with time even for the same experienced observer, and that this change can occur in a most unexpected manner. In order to analyze the homogeneity of a series of analogous observations, at least three such series are required. A sunspot series in addition to the Zurich series has been provided by the Soviet solar service, but the latter series only covers a period of slightly more than 20 years. The Freiburg series of Wolf numbers, on the other hand, is much shorter. Moreover, it was obtained by averaging the data of various observatories and thus differs essentially from the Zurich and Soviet relative-sunspot series.

Vitinskii's comparison of the Zurich and Soviet series of Wolf numbers has shown that either one of these series or both of them are not strictly homogeneous. A more exact decision concerning the homogeneity of these series cannot be made at present, since a sufficiently long third series is still not available. Nevertheless, the above factor is an indication that conversions from the Soviet system to the Zurich system can be made only with great caution. This point deserves special attention, since most of the cyclic regularities which are of significance for solar-activity forecasting have been obtained on the basis of the data of the Zurich system, which is considered as the international system.

Since the Zurich series of Wolf numbers covers over 200 years and shows a high correlation with various geophysical indexes (especially ionospheric characteristics), it is very important that this series be continued. At the same time, it would be very interesting to consider certain problems which might assist an evaluation of the reliability of the Zurich numbers. Some of these problems are: criteria for the discrimination of sunspot groups, the effect of the visibility conditions on Wolf-number determination, and the influence which the limb effect has on these determinations.

The last problem has been considered by several authors, in particular by Gleissberg et al. These studies have shown that the limb effect acts differently on groups with different "populations" (with different numbers of sunspots). Consequently, the central zone is the most reliable for sunspot counting. To the best of our knowledge, no adequate studies of the effect of visibility conditions on Wolf-number determination have been made so far, and such studies will be impossible unless the atmospheric conditions accompanying sunspot observations are recorded.

It is clear from the preceding discussion that daily Wolf numbers are not very significant. However, monthly and yearly relative sunspot numbers are quite suitable both for comparison with the various geophysical indexes and for forecasting. This is particularly true for the yearly values. For monthly Wolf numbers the instability of coefficient k may produce deviations of 20 to 25% in either direction.

Table I of the Appendix lists the Zurich Wolf numbers for each month from 1749 to 1961. In the following, these data will be referred to as the observed numbers, since they have been obtained by a simple averaging of the values observed during a month or during a year, as contrasted with the smoothed values of the relative sunspot numbers.

The Wolf numbers are generally smoothed according to the formula

$$W_i = \frac{1}{2}\left(\frac{W_{i-6} + W_{i-5} + \ldots + W_{i+5}}{12} + \frac{W_{i-5} + W_{i-4} + \ldots + W_{i-6}}{12}\right). \qquad (2)$$

This formula is used in order to eliminate the effects of terrestrial atmospheric conditions. Some authors, such as Vsekhsvyatskii, have suggested that an annual variation of the Wolf numbers exists. However, this proposition has recently been questioned. Nevertheless, since many forecasting regularities have been determined on the basis of smoothed relative sunspot numbers, we will list the values of these numbers for each month from 1749 to 1960 in Table II of the Appendix.

The definite subjectivity of the Wolf numbers led some researchers studying the ionosphere to try to avoid solar indexes describing the ultraviolet radiation of the sun and to substitute ionospheric solar-activity indexes for them. Not only does this series cover a period of less than 20 years, but also such an attempt itself represents a kind of self-deception, since ionospheric indexes are also influenced by the various effects characterizing the terrestrial atmosphere, so that one of the most essential difficulties has not been overcome.

Finally, let us mention briefly Kopecký's interpretation of the Wolf numbers. This is of more significance, in that two new indexes are introduced, namely the theoretical frequency of occurrence of spot groups f_0, and the average theoretical group lifetime T_0. These indexes will not be considered here in detail, but it should be noted that they were obtained from the Greenwich data using the spot groups appearing in the central zone and assuming uniform distribution of groups over the longitude. Kopecký showed that $W \sim f_0 T_0$, and that the first factor f_0 exhibits almost perfect 11-year periodicity, while the second factor T_0 does not. Consequently, it follows that the Wolf numbers reflect two different processes, which appear to be superposed on one another.

Before going on to the second fundamental index of solar activity, the sunspot-group area, it should be noted that the Wolf numbers are determined only for the visible hemisphere of the sun. Therefore, when these numbers are used it is tacitly assumed that an analogous spot pattern exists on the unobservable hemisphere as well.

Recently, attempts have been made to obtain what are known as global relative sunspot numbers. Becker and Kiepenheuer used visibility functions which they derived for various types of spot groups in order to plot the curves for group development, and they read from these curves the spot numbers during the 14-day period when the spots were invisible. For groups with short lifetimes, different probability assumptions had to be made. Then, when the daily results were added up, they obtained global sunspot numbers, these being the number of spots on the entire solar surface. However, since this procedure is quite difficult and since the global numbers are purely hypothetical quantities, therefore they have not been used in practice up to the present.

The spot-group areas, which will now be considered, are also determined only for the visible hemisphere of the sun. This index of solar activity was first suggested by Carrington in Greenwich in 1874. In contrast to the Wolf numbers, which are determined both photographically and visually, the spot areas are measured only from photographs. All the areas measured are referred to the solar center.

The spot areas are usually given in millionth parts of the solar disk or in millionth parts of the visible solar hemisphere. In the latter case the curvature of the solar surface is taken into consideration. It should be noted that in general the area determined is that of umbra plus penumbra, and this is the index which has become the most popular for practical applications. Occasionally, however, a study of the development of individual groups is based on the areas of the spot nuclei, which are measured at Greenwich. Finally, the area of the largest spot in a group, a quantity which is determined by the stations of the Soviet solar service, can also be used for this purpose.

The spot area represents a more objective index than the Wolf numbers. However, in the first place, the spot areas apparently reflect the other component of solar radiation, the corpuscular component, while, in the second place, the spot-area series is less than one-half as long as the spot-number series. Therefore, with due allowance to the advantages of this index, its forecasting value is much lower.

In one case, however, sunspot areas represent indispensible indexes, this case being when the asymmetry of sunspot-formation activity in the northern and southern hemispheres is studied. This is true because Wolf numbers are found separately for the two hemispheres only within one 11-year cycle, while spot areas have been determined continuously [for the two hemispheres] from 1874 to the present. This makes it possible, on the basis of certain features of sunspot-area asymmetry, to arrive at some interesting forecasting conclusions.

We have already mentioned that the Greenwich series is the longest existing series of sunspot-group areas. Sunspot areas are also determined by the Soviet solar service, and Vitinskii's comparison of the two series indicates that they are homogeneous and that thus conversion from one series to the other does not involve any special difficulty.

With all its advantages, the spot-area system has two essential defects, and these also apply to the Wolf numbers. First, the areas are influenced by the visibility conditions and, second, they are influenced by the limb effect. Since in the case of spot areas photographs are made, thus a change in photo contrast cannot but influence the accuracy of area measurements for individual spots, especially in the penumbra. If the size of the disk, which depends on the particular instrument used, is not large enough, then under poor atmospheric conditions some of the smaller spots may even be lost. Sunspot areas near the solar limb can be measured only with a high degree of uncertainty, so that when determining the group areas in this region (in millionths of the solar hemisphere), where sec ρ is very large, the measurement errors may be quite considerable.

Thus, monthly and yearly averages are preferred to the daily values of the total spot area for the entire disk. In the following, the word "total" will not actually be used, but it will be the total area which is implied. In addition to the sunspot areas for the entire solar disk, spot areas for the northern and southern hemispheres, generally for one rotation of the sun, are also introduced occasionally.

There is a definite statistical correlation between the Wolf numbers W and the sunspot areas S (the correlation coefficient is about +0.85). On the average, this correlation may be expressed as

$$S = 16.7W. \tag{3}$$

Let us now consider briefly some other solar-activity indexes which are closely related to the sunspot areas. In their study of certain special features of spot-group development during the various phases of the 11-year solar cycle, Eigenson and Mandrykina used the average maximum area $\bar{S}_M$ of the spot groups as an index. This index is closely related to Kopecký's quantity T_0. It should be noted, however, that in some cases the determination of S_M for individual spot groups is very unreliable, and this is particularly true when the maximum area is attained near the solar limb or in the unobservable hemisphere of the sun. Nevertheless, this uncertainty is not greater than that for the index S, and thus there is no reason why the index $\bar{S}_M$ should be rejected. The index $\bar{S}$, the average area of a sunspot group, is also sometimes used for certain comparisons with geophysical phenomena. Since $\bar{S}_M$ and $\bar{S}$ have no particular prognostic value (at least, as far as the sun is concerned), we will not discuss these indexes further.

At present, numerous solar indexes characterizing various phenomena in the different layers of the solar atmosphere are available. Naturally, it would be highly desirable to use these indexes to develop methods for solar-activity prediction. However, unfortunately, the series of data referring to these indexes are rather short and, what is particularly significant, discontinuous. Moreover, there is also serious doubt as to their homogeneity. Because of these factors we are forced to disregard this whole set of solar indexes, and only to refer to them occasionally for purely qualitative evaluations of the situation.

In summarizing all the preceding discussion of solar-activity indexes, it must be conceded that at present we actually have nothing better than empiricostatistical methods of Wolf-number prediction. The forecasting of sunspot areas is mostly confined just to qualitative estimates. Moreover, this situation also determines who can derive the main benefit from solar forecasts. Now such forecasts are mainly used by radiophysicists. Meteorologists and oceanologists, on the other hand, use Wolf-number forecasts only for certain qualitative estimates. This book will discuss these empiricostatistical methods for forecasting solar activity, especially methods for forecasting the Wolf numbers.

Solar-activity forecasts can be divided into three groups: 1) short-range, 2) long-range, and 3) ultralong-range.

Short-range forecasts have as their goal the calculation of certain solar indexes several days in advance (a period less than one solar rotation). This represents the most complicated problem, and so far no satisfactory methods for short-range forecasting have been developed. Therefore, we are restricted to a discussion of long-range-forecasting methods only (including long-range and ultralong-range forecasts, as defined below). It should be noted that the division into long-range and ultralong-range forecasts is rather arbitrary, and that it was introduced due to differences in the respective methods as they stand at present.

Long-range forecasts of solar activity include all forecasts referring to the time included in one cycle. It is convenient to divide this group into two subgroups, namely medium-period and long-period forecasts. Medium-period forecasts include monthly and quarterly forecasts. This subgroup has begun to be developed only during recent years; it is not distinguished by any variety in methods, and the accuracy of medium-period forecasts is correspondingly low. The main difficulty in developing medium-period forecasts is that here fluctuational processes are involved. Although some methods have been worked out for predicting the evolution of a fluctuation, still it is virtually impossible at present to foresee the appearance of a fluctuation. Consequently, at times when powerful fluctuations arise, such forecasts have especially high errors.

The second subgroup of long-range forecasts of solar activity includes forecasts made a year or several years in advance. Some progress has been made with respect to such predictions, particularly once the so-called "superposition hypothesis" was refuted, a step which stimulated the detailed study of intracycle regularities. Forecasts made a year or several years in advance now have a fairly high accuracy. The fact that several different methods for making such forecasts have been developed is a considerable advantage, since the defects of the various methods appear to compensate for one another.

Ultralong-range forecasts predict the situation over the next 11-year solar cycle or over several future cycles. Such forecasts have always attracted the attention of solar scientists, but have on the whole resulted more often in failures than in successes. The reason for this is that regular telescopic observations of the sun have been made for only a little more than 200 years, so that many higher-order cyclic regularities have probably remained unknown to us. The difficulty is increased still more by the fact that solar activity is not strictly periodic, since it is affected by a multiplicity of perturbations. Nevertheless, some success has been achieved in this sphere during the last decade.

In this book the above classification of solar-activity forecasts has been adhered to, and the text has been divided as follows. Chapter I discusses the principal regularities of solar activity, which will be made use of directly in the subsequent discussion. Chapter II deals with long-range forecasts made a year or several years in advance. Chapter III discusses medium-period long-range forecasts. Finally, Chapters IV and V deal with methods of ultralong-range forecasting.

Naturally, this book cannot pretend to be an exhaustive presentation of all existing methods of solar-activity forecasting. An attempt has been made to select only the most important of these methods, on the assumption that this information will be useful to heliophysicists, geophysicists, and also to any others interested in sun-earth problems.

Finally, let us note too that some progress has recently been made toward developing a theoretical method for forecasting the solar indexes. In particular, Rubashev has made a contribution which is of unquestionable interest in this respect. Let us hope that in the near future these studies will yield tangible results and that solar-activity forecasts will be placed on a firm physical basis.

Chapter I

THE BASIC REGULARITIES OF SOLAR CYCLES

§ 1. General Remarks

Solar activity has attracted the attention of many investigators from ancient times until the present. The first observations were fragmentary and made with the naked eye, but later, from the time (1610) when Galileo used the telescope for solar observations, the studies became more and more regular. However, almost 150 years still had to pass before the first important regularities of solar activity were derived from these observations.

Since in the following the main emphasis will be on forecasts of the various indexes connected with sunspots, this chapter will be devoted to a discussion of the principal regularities in sunspot activity. Many other active solar formations, such as faculae, chromospheric flares, filaments, prominences, regions of increased coronal-line emission, and radiospots, are all mutually related, and they appear to constitute one single complex of solar activity. The U and BM magnetic regions can also be included in this complex.

If the magnetic regions are disregarded, sunspots constitute the cores of the so-called active regions, or active centers, of the sun. It is the presence of active solar regions which determines the actual level of solar activity. We will consider sunspot activity from two aspects, temporal and spatial. The temporal behavior of sunspots is particularly significant for our purposes. Spatial considerations can also be employed in some forecasting procedures, and at any rate these are closely related to the temporal behavior of sunspots. In the following, therefore, the term "solar activity" will refer to sunspot activity only.

§ 2. The Schwabe-Wolf Law

The very first regular observations of sunspots showed that the number of spots varies with time. This was first observed by the Danish astronomer Horrebow in the 1770's, on the basis of his solar observations between 1761 and 1769. Unfortunately, most of Horrebow's data were lost in the shelling of Copenhagen during the Napoleonic wars, and his discoveries were forgotten. This fact became known only after Wolf had stated his law describing the variation of the sunspot numbers (Gleissberg, 1952).

In 1843, on the basis of 20 years of observations, the amateur astronomer Schwabe established that solar activity varies with a period of about 10 years (Schwabe, 1844). This led Rudolph Wolf, the director of the Zurich Observatory, to set up systematic observations of the variations in sunspot activity. These observations then led to the discovery of the 11-year sunspot cycle.

Wolf showed that the numbers of sunspots, or more precisely the Wolf numbers, are subject to cyclic fluctuations, with an average cycle duration of 11.1 years. This very important regularity in solar activity is generally called the Schwabe-Wolf law. It should be noted that this law is valid for other active solar formations as well. It has been found that the length of the sunspot cycle changes rather sharply from one cycle to another, and that it can vary from 7.3 to 17.1 years.

The part of the cycle in which the sunspot number goes through a minimum is called the epoch of sunspot minimum, while the part including the maximum is called the epoch of sunspot maximum. An increase in solar activity is represented by a rising curve and a decrease is represented by a descending curve.

As mentioned in the Introduction, the relative sunspot numbers have been regularly determined in Zurich since 1749. However, even the scanty observations made from 1610 to 1749 were sufficient to establish epochs of maxima and minima for the sunspot cycle. Later, these epochs were determined according to the variation of the smoothed monthly Wolf numbers.

TABLE 1

Epochs of extrema for 11-year sunspot cycle (Zurich data)

Epochs of minima				Epochs of maxima			
1610.8	1698.0	1784.7	1878.2	1615.5	1705.5	1788.1	1883.9
1619.0	1712.0	1798.3	1889.6	1626.0	1718.2	1805.2	1894.1
1634.0	1723.5	1810.6	1901.7	1639.7	1727.5	1816.4	1907.0
1645.0	1734.0	1823.3	1913.6	1649.0	1738.7	1829.9	1917.6
1655.0	1745.0	1833.9	1923.6	1660.0	1750.3	1837.2	1928.4
1666.0	1755.2	1843.5	1933.8	1675.0	1761.5	1848.1	1937.4
1679.5	1766.5	1856.0	1944.5	1685.0	1769.7	1860.1	1947.5
1689.5	1775.5	1867.2	1954.5	1693.0	1778.4	1870.6	1957.9

Table 1 shows the epochs of maxima and minima of the 11-year sunspot cycles, compiled on the basis of the Zurich data for the years from 1610 to 1957.

The epochs of extrema for the 11-year sunspot cycle given in Table 1 were determined to the nearest tenth of a year. These data have been used to derive various forecasting relations, and thus they are given here in their original form. It should be noted, however, that cyclic curves plotted from the monthly observed Wolf numbers are subject to strong fluctuations, and, as shown by Vitinskii, the length of these fluctuations ranges from 3 months to a year. Therefore, even the Zurich astronomers were forced in some cases to introduce corrections into the epoch values obtained from the smoothed monthly Wolf numbers. Chernosky has recently (1954) shown that smoothing over a 5-month period gives much more satisfactory results

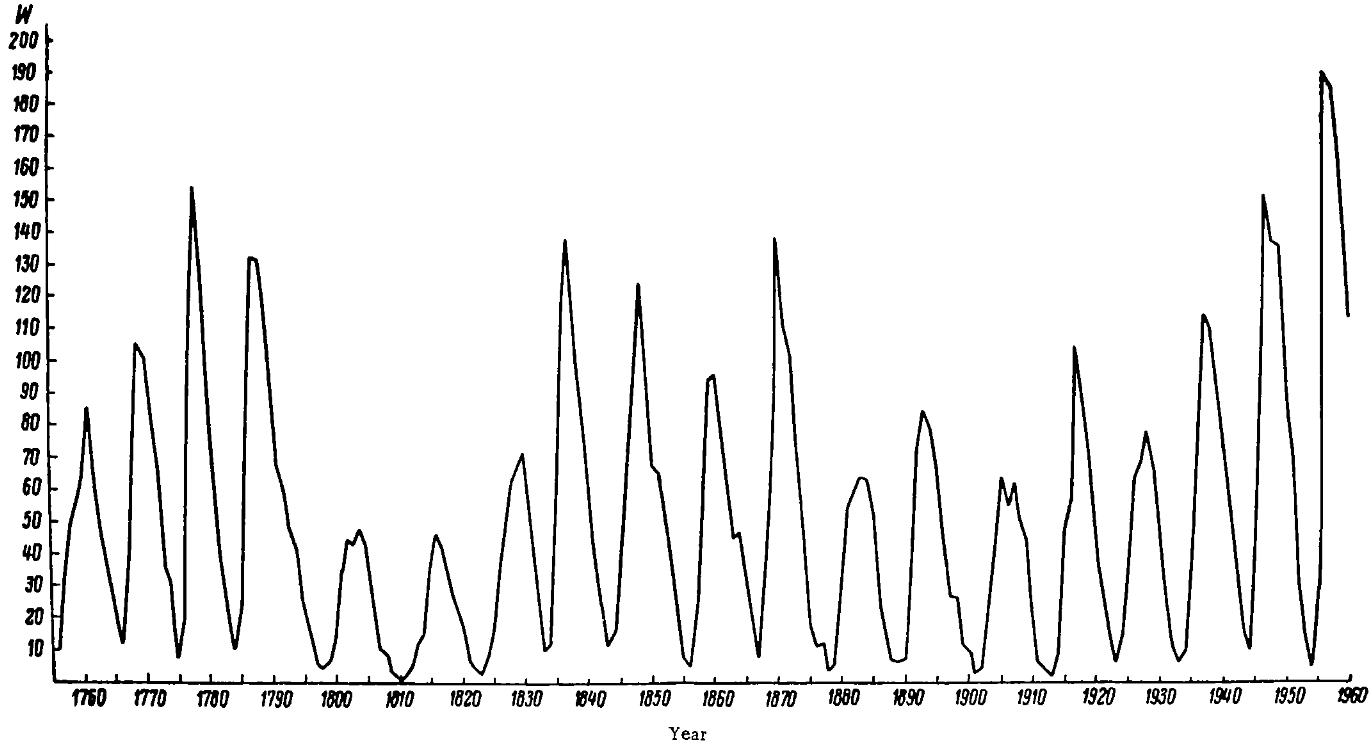
W
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
1760 1770 1780 1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 1930 1940 1950 1960
Year

FIGURE 1

than smoothing according to formula (2) in the Introduction. It is interesting, in this respect, that 5 months is also the average duration of the fluctuations in the relative sunspot numbers.

Taking all these factors into account, Vitinskii and Ikhsanov (1960) suggested determining the epochs of extrema for the 11-year cycle to the nearest 3 months, which is the minimum duration of a fluctuation, according to the observed quarterly Wolf numbers (listed in Table III of the Appendix). Two curves enveloping the peaks of the positive and negative fluctuations were used for this. If both curves go through a maximum, then this point corresponds to a maximum of the 11-year cycle, and the epoch of minimum is determined analogously. If at a given point the curves have opposite shapes, then the point represents only a strong fluctuation. Table 2 lists the epochs of extrema obtained using this method. The Roman numerals

TABLE 2

Epochs of extrema for 11-year sunspot cycle (according to Vitinskii and Ikhsanov)

Epochs of minima				Epochs of maxima			
II – 1755	II – 1810	I – 1867	I – 1924	III – 1761	I – 1817	II – 1870	III – 1928
III – 1766	II – 1823	I – 1879	IV – 1933	IV – 1769	II – 1829	I – 1884	III – 1937
III – 1775	II – 1834	IV – 1889	II – 1944	II – 1778	I – 1837	III – 1893	II – 1947
II – 1784	III – 1843	III – 1901	II – 1954	IV – 1788	IV – 1847	IV – 1905	IV – 1957
II – 1798	III – 1855	II – 1913		IV – 1804	III – 1859	III – 1917	

in Table 2 indicate the quarters of the year. Epochs of extrema have been given only for those 11-year solar cycles for which the Wolf numbers have been determined.

According to the Zurich enumeration for the 11-year sunspot cycles, the epoch of minimum for the 1st cycle occurred in 1755. Wolf numbers for more than 18 cycles are now available, and Figure 1 shows the curve of variation in yearly relative sunspot numbers for the 18 complete cycles and for the elapsed part of the 19th cycle. The curve shows that different 11-year solar cycles are characterized not only by different durations but also by different intensities, as determined by the maximum Wolf numbers.

The relation between the main parameters describing the 11-year sunspot cycle was studied in detail by Waldmeier (1935), during the development of his widely known "eruption" hypothesis of solar cyclicity, a theory which will now be discussed at some length.

Let us first introduce some additional notation. Let T denote the duration in years of the rising branch of the curve for the 11-year cycle, and let θ denote the duration of the descending branch, from the epoch of maximum to the year in which $W = 7.5$; in addition, let W_5 denote the Wolf number 5 years after the maximum of sunspot activity. Finally, let Σ_1 be the sum of the smoothed monthly relative sunspot numbers over the ascending branch, let Σ_2 be the sum of these numbers over the descending branch, and let W_M be the maximum monthly Wolf number. According to Waldmeier, we can now single out the following basic features of the sunspot curve.

a. The duration T of the ascending branch increases with a decrease in the height W_M of the maximum. For even cycles (according to the Zurich enumeration), this property can be expressed by the following relation:

$$\log W_M = 2.69 - 0.17T. \quad (1.1)$$
$$\pm 0.09 \pm 0.02$$

For odd cycles, we have the relation

$$\log W_M = 2.48 - 0.10T. \quad (1.2)$$
$$\pm 0.10 \pm 0.02$$

b. The quantity θ increases with W_M:

$$\theta = 3.0 + 0.030W_M. \quad (1.3)$$
$$\pm 0.6 \pm 0.006$$

c. For an epoch 5 years after the sunspot maximum, we have the relation

$$W_5 = -11.4 + 0.29W_M. \quad (1.4)$$
$$\pm 6.7 \pm 0.06$$

d. The statistical relations between the sums Σ_1 and Σ_2 and the maximum Wolf number W_M have the form

$$\Sigma_1 = 0.4W_M + 2538, \quad (1.5)$$
$$\pm 3.2 \quad \pm 340$$

$$\Sigma_2 = -572 + 40.6W_M. \quad (1.6)$$
$$\pm 600 \pm 5.9$$

e. Finally, it follows from formulas (1.1), (1.2), and (1.3) that the ratio $Q = T/\Theta$ for the even cycles is

$$Q = \frac{15.64 - 5.81 \log W_M}{3.0 + 0.030W_M}, \quad (1.7)$$

while for the odd cycles it is

$$Q = \frac{24.8 - 10.00 \log W_M}{3.0 + 0.030W_M}. \quad (1.8)$$

The values of Q for different cycles range from 0.37 to 1.72, with an average of 0.7. The rise of the cyclic curve is more rapid than the descent only for cycles with average and high maximum relative sunspot numbers. For cycles with low W_M the reverse seems to be true.

The above relations make it evident that the behavior of the cyclic sunspot curve is determined mainly by the parameter W_M. Consequently, the time variation in sunspot activity $F(t)$ has the form

$$F(t) = F(t, W_M). \quad (1.9)$$

Stewart and Panofsky (1938) used Waldmeier's assumption that each cycle can be considered as an individual eruption as the basis for an empirical formula for $F(t)$. They demonstrated that the variation of the cyclic curve of relative sunspot numbers is approximated quite well by the expression

$$W = F\theta^a e^{-b\theta}, \quad (1.10)$$

where θ is the time interval in years between the given instant and the epoch of minimum, while a, b, and F are constant for a given solar cycle. The quantity F essentially represents the scale of the Wolf-number time base.

In deriving formula (1.10), Stewart and Panofsky used the Zurich data for the epochs of minima and maxima, for the maximum smoothed monthly

Wolf numbers, and for the yearly observed Wolf numbers, over 16 sunspot cycles. The constants a, b, and F were determined by different methods and were found to vary both according to the method used and according to the cycle considered. It will not be necessary to discuss here the methods employed; it will be enough just to give the table for the values of a, b, and $\log F$ compiled by these authors. Let us note that in some cases a and b are not given, since the figures obtained were meaningless.

TABLE 3

Main parameters of Stewart-Panofsky formula, for 16 solar cycles

Cycle number	1st method		2nd method			3rd method		4th and 5th methods	
	a	b	a	b	$\log F$	a	b	a	b
1	10.43	1.51	7.89	1.32	-0.763				
2	3.44	1.03	3.75	1.07	+1.650	2.71	0.92	3.08	0.92
3	3.99	1.32	5.95	1.60	+1.390	1.20	0.76	2.30	0.76
4	2.41	0.68	3.35	0.79	+1.500	1.32	0.51	1.82	0.51
5	11.66	1.53	6.21	1.12	-0.233				
6	10.80	1.71	12.30	1.83	-3.147	3.74	1.02	6.44	1.02
7	11.22	1.55	7.55	1.28	-0.682				
8	3.97	1.15	4.86	1.27	+1.447	2.13	0.85	2.94	0.85
9	5.63	1.14	7.13	1.28	-0.103	2.06	0.71	3.49	0.71
10	3.97	0.91	4.88	1.01	+0.772	2.10	0.68	2.96	0.68
11	4.29	1.20	5.73	1.38	+1.095	1.61	0.76	2.71	0.76
12	7.07	1.31	4.62	1.07	+0.944				
13	5.38	1.12	4.95	1.08	+0.823	8.10	1.37	6.58	1.37
14	4.88	0.97	4.14	0.89	+0.839	10.48	1.41	7.09	1.41
15	6.54	1.54	7.04	1.60	+0.550	4.36	1.27	5.38	1.27
16	6.20	1.20	3.92	0.96	+1.198				

Table 3 (2nd method) indicates that b and $\log F$ can be expressed quite well in terms of a as follows:

$$b = 1.60 \log a + 0.03, \tag{1.11}$$

$$\log F = -0.537a + 3.63. \tag{1.12}$$

Later authors who have attempted to find an analytical form for the cyclic curve have mostly used expressions such as (1.10). In particular, Gleissberg (1951a) gives an average curve of this type, with $a = 7.1832$ and $b = 1.2013$. Further investigations, however, caused Gleissberg to divide all the solar cycles into three groups, in terms of intensity, and then to determine constants a and b separately for each of these groups.

Granger recently (1957) devised a statistical model describing sunspot activity. In this model he showed that the cyclic curve can be represented quite accurately by the formula

$$W = f(x)[g(x) + \varepsilon_x], \tag{1.13}$$

where $f(x)$ is an amplitude factor which probably contains a long-period fluctuation, and $g(x)$ is a curve which may be approximated roughly by the equation

$$g = \theta^{2.67} e^{1.73 - 0.63\theta}, \tag{1.14}$$

repeated for different intervals. The term ε_v is a random function which is on the average zero. Here, equation (1.14) represents a modification of formula (1.10).

If we take into account that in the Stewart-Panofsky formula the quantity F is the scale of the time base for the Wolf number, then we have here a two-parameter formula. Since a and b are not independent of one another, some authors have tried to replace this relation by a one-parameter formula (see, for example, Thüring, 1955). However, there are many properties of solar cyclicity which do not back up this, at first glance so attractive, simplification. The problem is that the processes corresponding to the rising part of the solar-cycle curve differ somewhat markedly from the processes corresponding to the descending part of the curve. This is illustrated by the fact that the correlations established for the rising part of the curve are much better than those for the descending part. In the subsequent discussion some other features of sunspot activity will be considered which support this statement.

The studies of Xanthakis (1959) have shown that some of the properties of the sunspot cycle depend on the length of the rising part of the curve. It is apparently precisely this parameter that characterizes the shape of the cycle curve, the maximum height, the fluctuation pattern, etc. The descending part of the curve, on the other hand, seems to correspond to a process which depends on the properties of the higher layers of the subphotosphere.

Gleissberg (1949) tried to give a physical explanation of the Stewart-Panofsky formula. He concluded that the rate of formation of new spots and the rate of decay of existing spots are both proportional to the intensity of solar activity. Moreover, he decided that the rate of formation of new spots is inversely proportional to the time elapsed after the beginning of the cycle, whereas the rate of sunspot decay is independent of time. Such an explanation can hardly be accepted. Although the rate of formation of new spots may possibly be proportional to the intensity of solar activity, still this does not apply to the rate of variation of existing sunspots. This is evident, for instance, from the fact that the product of the sunspot-group frequency times the average group lifetime is proportional to the Wolf number (Kopecký, 1959).

The Schwabe-Wolf law can be formulated either for the entire sun or for its northern and southern hemispheres separately. However, this will not be gone into at this point, but will be reserved for the discussion, later in the book, of the asymmetry of solar activity in the northern and southern hemispheres. It should be observed, though, that the formulation of the Schwabe-Wolf law is almost the same for the two solar hemispheres, and differs only in certain minor details. At present, however, this is not important.

Finally, it should be mentioned that the Schwabe-Wolf law applies not only to sunspot activity but to other types of solar activity as well. This is equally true of another basic law describing solar activity, namely Spörer's law, which will be discussed in the next section.

§ 3. Spörer's Law

Series of sunspot observations extending over many years have shown that most sunspots appear between latitudes from ±45° to ±5°. Outside of these regions, sunspots have been observed only very rarely, and these have been mainly pores. The maximum latitude in which a pore has been recorded is 71°. On the average, the width of the sunspot zone is about 20°.

The studies of Carrington (1858), and later the independent studies of Spörer (1881) and Maunder (1917), showed that the entire sunspot zone shifts during the cycle from higher to lower heliographic latitudes. Usually, the first sunspot groups in a given solar cycle appear at latitudes of about 30°, and at the end of the cycle the average latitude of the spot groups is about 8°. Table 4 gives the Greenwich data for the average yearly heliographic latitudes of sunspots from 1878 to 1953.

TABLE 4

Average yearly heliographic latitudes of sunspots from 1878 to 1953 (Greenwich data)

Year	φ	Year	φ	Year	φ	Year	φ	Year	φ	Year	φ
1878	7°.58	1891	20°.31	1904	16°.57	1917	14°.63	1930	9°.87	1943	10°.09
1879	21.96	1892	18.39	1905	13.10	1918	12.75	1931	8.31	1944	21.53
1880	19.64	1893	14.49	1906	13.99	1919	10.76	1932	8.32	1945	20.22
1881	18.30	1894	14.18	1907	12.12	1920	10.43	1933	10.56	1946	20.00
1882	17.81	1895	13.54	1908	10.38	1921	7.90	1934	23.75	1947	17.38
1883	13.06	1896	14.33	1909	9.71	1922	8.02	1935	23.30	1948	14.19
1884	11.26	1897	7.96	1910	10.53	1923	15.26	1936	20.35	1949	13.33
1885	11.77	1898	10.49	1911	6.49	1924	22.73	1937	17.02	1950	13.41
1886	10.38	1899	9.54	1912	8.06	1925	20.20	1938	14.79	1951	11.32
1887	8.44	1900	7.74	1913	23.23	1926	18.66	1939	13.42	1952	8.00
1888	7.39	1901	10.37	1914	21.79	1927	15.05	1940	11.17	1953	9.86
1889	11.61	1902	17.64	1915	18.77	1928	13.50	1941	10.38		
1890	21.99	1903	19.94	1916	15.81	1929	10.51	1942	8.99		

The data in Table 4 show that the rate of drift of a sunspot zone changes during the cycle. On the average, this rate is highest during the increasing part of the cycle and then it gradually decreases. In the epoch of sunspot maximum the average latitude of a sunspot zone is about 15°.

Spörer's law is best illustrated using the "butterfly" diagrams of Maunder, which show the latitudes of all the sunspots, regardless of their size, plotted on the corresponding time scale. Figure 2 shows the butterfly diagrams for the 12th through the 18th solar cycles (Zurich enumeration). In addition to showing the drift of the sunspot zones, the butterfly diagrams make it clear that low-latitude spots from the previous cycle are generally observed near the epoch of sunspot minimum, simultaneously with high-latitude sunspots of the new cycle. Consequently, the actual length of a solar cycle is somewhat greater than the time between successive epochs of minimum.

It should be noted, by the way, that it is rather difficult to differentiate between the old and new sunspots in the epoch of minimum. However, such a differentiation can be made quite reliably according to the magnetic polarity of the spots, since, as Hale and Nicholson (1938) have shown, this polarity changes from cycle to cycle.

Gnevyshev (1944) showed that the Spörer curves can be translated along the time axis in such a way that they will all coincide approximately with one another. The scatter of points about the mean, on the curves of different cycles, is about 1°, a value which is within the margin of error for determining the sunspot coordinates. This factor can be utilized for forecasting purposes.

Gleissberg (1958) studied the variation of the width of the sunspot zone during the solar cycle. On the basis of an examination of the outer points on the butterfly diagrams, he found that on the average the width of the sunspot region varies, concomitantly with the cyclic curve, from 8° in the epoch of minimum to 36° in the epoch of maximum. This being the case, it is hardly possible to speak of a simple drift of the sunspot zone toward the solar equator.

According to Kopecký (1958), spot groups appear at high heliographic latitudes mostly in the epoch of high sunspot maxima, rather than at the beginning of the 11-year cycle. This result is consistent with Gleissberg's conclusions concerning the latitudinal width of the sunspot zone.

Recently, Becker (1959) has shown that a second, high-latitude, sunspot zone exists. Whereas the first sunspot zone drifts during the cycle from higher to lower latitudes, the second zone gradually moves toward higher latitudes during the period between the epochs of minimum and maximum. In his study of this zone, Becker used the average spot area as the main parameter describing sunspot activity. This index was determined for individual latitude zones and years, and also for the entire cycle. The high-latitude sunspot zone could be distinguished when isoline charts for the index $\frac{1}{n}\sum_i \bar{S}_i \Big/ \frac{1}{N}\sum_j \sum_i \bar{S}_i$ were plotted. Here $\bar{S}_i$ is the average sunspot-group area, n is the number of groups in a given latitudinal zone in a given year, and N is the number of groups in a given cycle in all the latitudinal zones.

It should be mentioned here that the possibility of the existence of a second, high-latitude, sunspot zone had already been hinted at in the previously cited studies of Gleissberg and Kopecký.

Kopecký (1960) has shown that Maunder's butterfly diagrams and Becker's isoline charts represent two physically different aspects of solar activity. Whereas Maunder's diagrams indicate the frequency of occurrence of sunspot groups, Becker's diagrams show the average intensity of the groups. Since this is the case, it is difficult to agree with Becker's statement that Maunder's butterfly diagrams are more schematic than the Becker diagrams.

In the same article Kopecký verifies the actual existence of the second, high-latitude, sunspot zone, using as an index the daily average spot area. The latter is defined as $\Sigma S_i/N$, where ΣS_i is the total area of the sunspot groups, and N is the number of groups in a given day. Kopecký also demonstrates that the use of this index makes it possible to distinguish the second sunspot zone more clearly.

In his most recent study of this subject, however, Kopecký (1962) concludes that, because of Becker's somewhat biased approach to the data, the existence of the second, high-latitude, sunspot zone cannot yet be considered as definitely proven.

Finally, let us also mention that Lockyer's explanation (1904), according to which there is no single sunspot zone but rather several centers which migrate toward the solar equator during the 11-year cycle, has

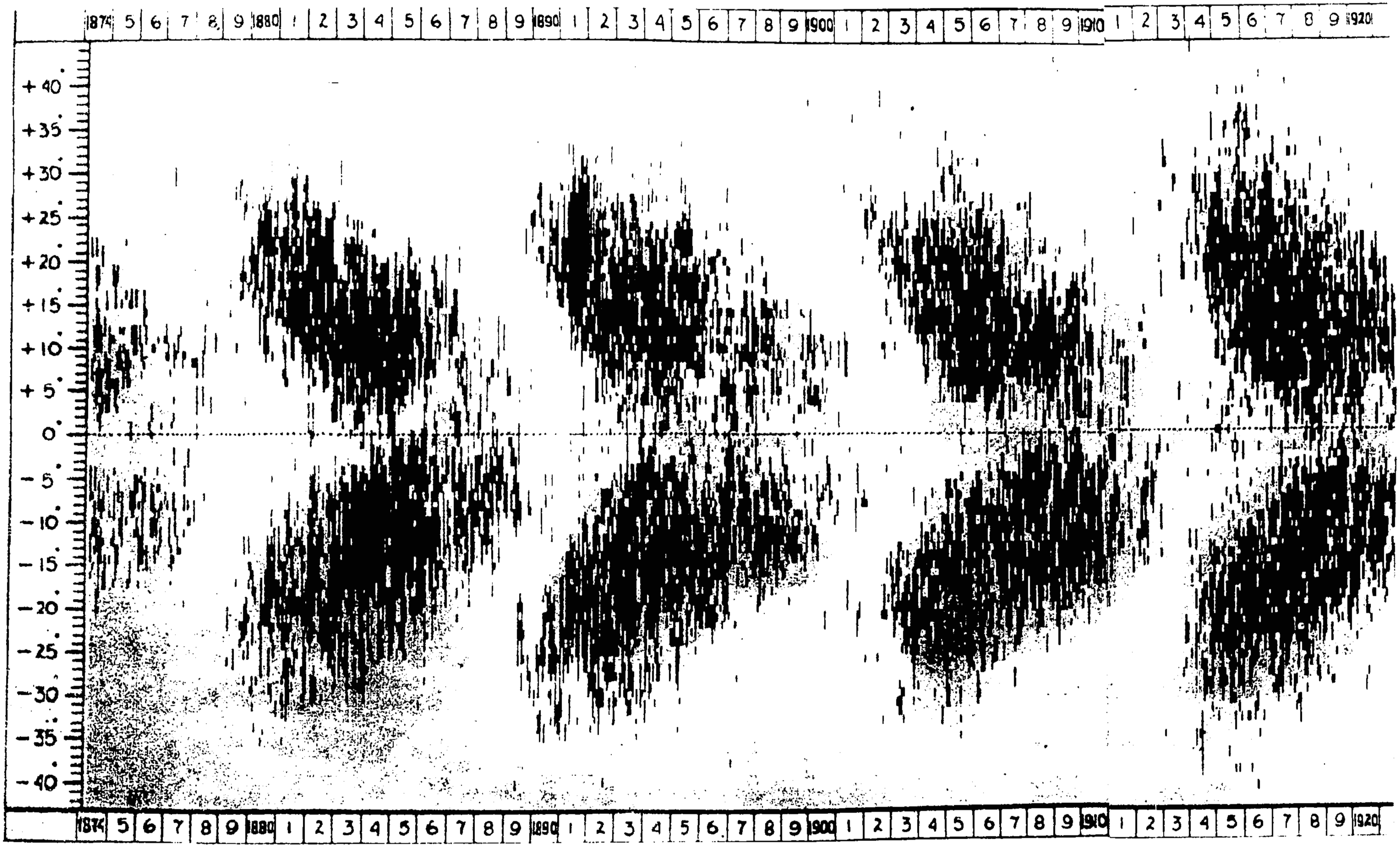

FIGURE 2. The butterfly diagrams

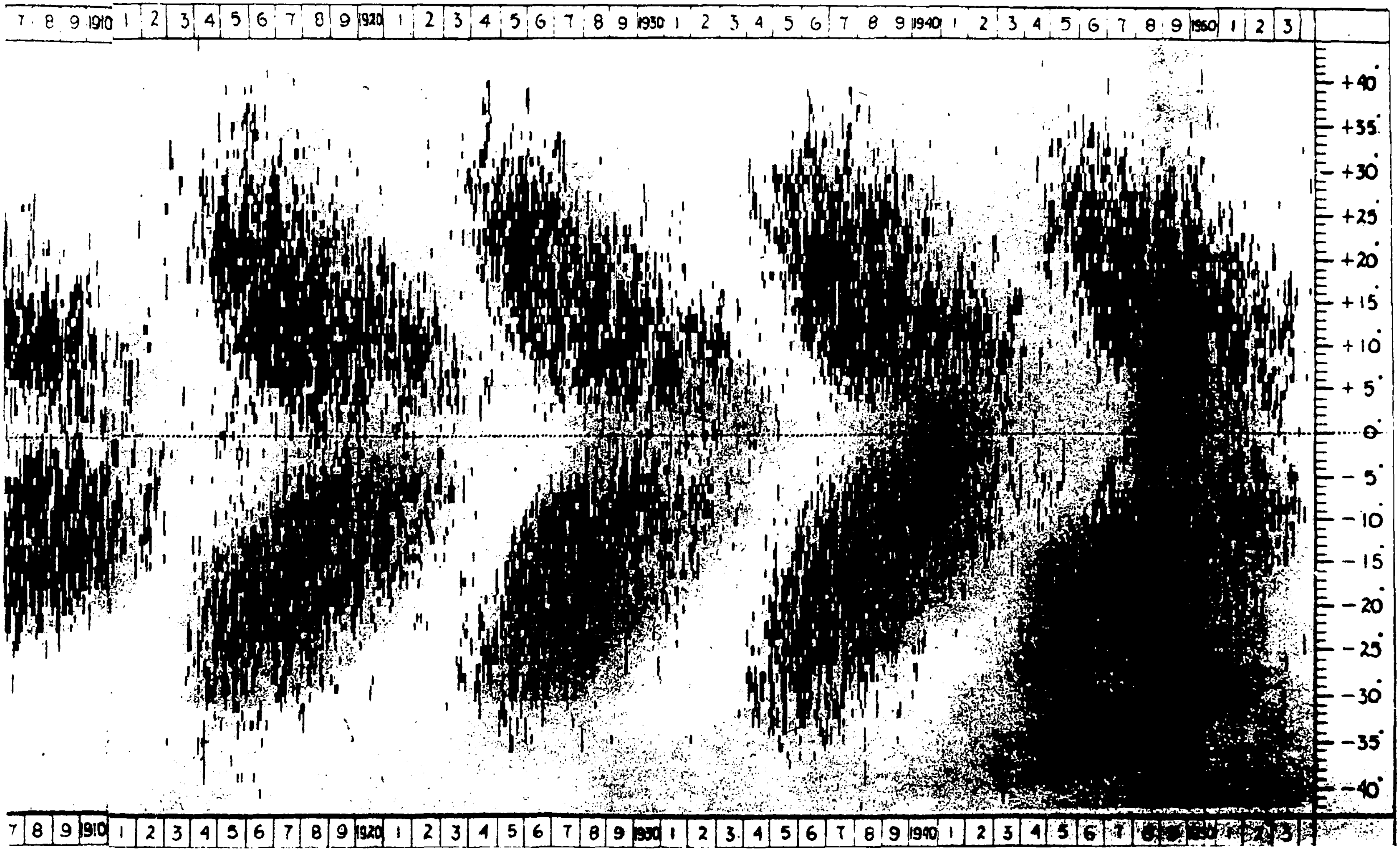

FIGURE 2. The butterfly diagrams

Job 1400/16-17

recently been revived. Bell (1960) studied the magnetic fields of sunspots using data from Mount Wilson and investigated the latitudinal distribution of sunspot groups in order to obtain a multizonal pattern decribing sunspot formation, on which the maxima of these zones were separated from one another by about 5° of latitude. According to Bell, these subzones do not shift toward the equator but rather appear sometimes and disappear sometimes. This explanation has been represented graphically by Bell by means of a "caterpillar" diagram (Figure 3), which suggests comparison with Maunder's butterfly diagram. It should be observed, however, that a division of the solar disk into 2° zones, even when the results obtained are smoothed, may lead to false conclusions, since in most cases the spot groups extend over more than 2° of latitude. Although Bell's results appear very attractive from the viewpoint of Alfvén's theory (1952), nevertheless no arguments against the objection just raised have yet been found. Vitinskii (1961d) has shown that the "caterpillar" diagram represents a fictitious result, obtained as a result of dividing the solar disk into latitudinal zones which are narrower than the latitudinal width of average-size sunspot groups and as a result of combining spot groups of different magnetic types arbitrarily. In addition, Eigenson has stressed the insufficient statistical substantiation of Bell's results, a factor which also renders her conclusions unacceptable.

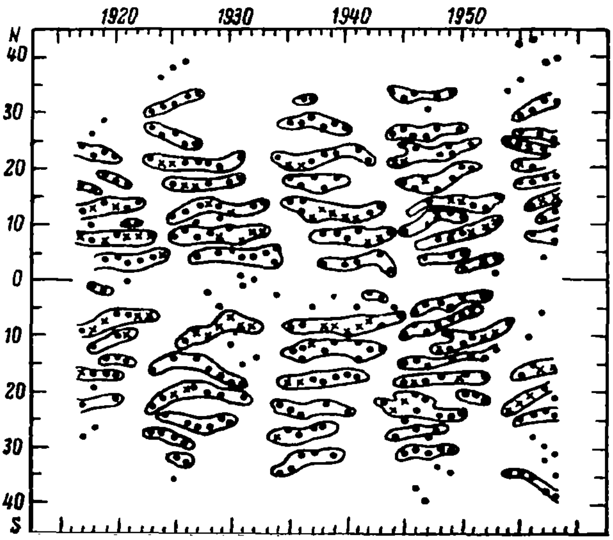

FIGURE 3

In conclusion, let us observe that Spörer's law has a somewhat different form for the northern and southern solar hemispheres. However, this problem will be discussed in detail below, in connection with the asymmetry of solar activity.

Thus, we have considered two basic regularities of solar activity. The Schwabe-Wolf law and Spörer's law. The first law describes the variation in relative sunspot number, and in other solar indexes, and provides a quite sharp definition of the epoch of sunspot maximum and a less sharp definition of the epoch of sunspot minimum. It is represented by a continuous function. The second law reflects the change in the average latitude of sunspots and other active solar formations, and it defines quite sharply the epoch of minimum but gives practically no indication at all of the epoch of maximum. This law is represented by a discontinuous function of time, in which the discontinuities correspond to the epochs of sunspot minimum for each cycle.

The relation between the Schwabe-Wolf law and Spörer's law is very important, and it has particular significance with respect to the forecasting of solar activity.

§ 4. The Correlation between the Schwabe-Wolf Law and Spörer's Law

The shifting of the sunspot zone as a function of the variation in Wolf number was studied by Waldmeier (1939), who used the data for the years 1836 through 1933, that is, for 7 solar cycles. If W_M is the maximum smoothed monthly Wolf number in a given cycle, φ_{-50} is the average smoothed heliographic latitude of the sunspot zone for the phase 50 solar rotations prior to the epoch of sunspot maximum, φ_M is this latitude in the epoch of maximum, and φ_{+50} is this latitude for the phase 50 solar rotations after the maximum, then according to Waldmeier the following relations are satisfied:

$$\varphi_{-50} = (17.58 \pm 1.74) + (0.0839 \pm 0.0189)\, W_M, \tag{1.15}$$

$$\varphi_M = (8.19 \pm 1.36) + (0.0699 \pm 0.0143)\, W_M, \tag{1.16}$$

$$\varphi_{+50} = (5.44 \pm 0.85) + (0.0427 \pm 0.0089)\, W_M. \tag{1.17}$$

These relations show that the sunspot zone reaches higher latitudes, the greater is the maximum relative sunspot number for the given cycle.

The correlation between the Schwabe-Wolf law and Spörer's law was studied in detail by Gnevyshev and Gnevysheva (1949). In their approach to this problem, these authors smoothed the yearly average latitudes of the sunspot zones for eight different 11-year cycles (1856 — 1943). In order to do this, they used the average curve for shifting of the zone, obtained by translating the Spörer curves for the different solar cycles until the best fit was found. This curve is shown in Figure 4, where the points indicate the individual determinations of φ. By measuring the time intervals between points on Spörer's curve for the given cycle and points on the average curve in Figure 4 (these points corresponding to the same values of φ) and then taking the average of these intervals, we will obtain the distance along the time axis over which the average curve for $\varphi(t)$ must be translated in order to give the best fit with the $\varphi(t)$ curve for the given cycle. In this way, the smoothed latitude values $\bar{\varphi}$ listed in Table 5 were obtained.

After studying the values of W for corresponding latitudes in different cycles, arranged according to the total "cycle intensity" ΣW (excluding the minimum years), Gnevyshev and Gnevysheva came to the following conclusions.

1. The change in W from cycle to cycle, at all latitudes higher than 14°, is proportional to ΣW. This follows from the fact that the values of the correlation coefficients between W for a given latitude and ΣW are higher. The values of W at latitudes of 12° and below are independent of ΣW, the correlation coefficients between W and ΣW being low.

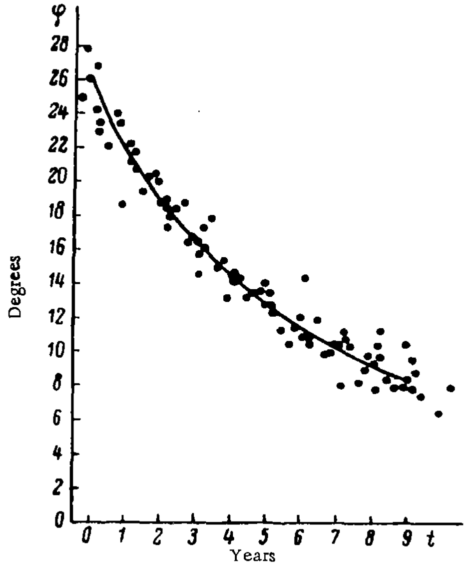

FIGURE 4

2. The standard deviations from the mean W for latitudes of 12° and below are small, whereas for $\varphi \geqslant 14°$ these deviations are large.

3. The more ΣW increases, the higher will be the latitude φ at which the maximum W in the given cycle is attained.

The first two conclusions imply that on the descending parts of the sunspot-cycle curves (that is, at low heliographic latitudes) W depends entirely on the latitude, and that this dependence is the same for all cycles. Consequently, if W is known, the average sunspot latitude at the year of sunspot maximum can be determined. The correlation between φ_M and ΣW is very high ($r = \pm 0.94$) and is given by the expression

$$\varphi_M = 7.^{\circ}4 + 0.^{\circ}016\Sigma W. \tag{1.18}$$

This relation gives the latitude at the year of sunspot maximum with a standard deviation of $\pm 0^{\circ}.9$, a value which is within the allowable error for measuring the sunspot coordinates.

The following conclusion can be drawn from a study of the relation between φ_M and ΣW: the greater ΣW is, the earlier the epoch of sunspot maximum for the solar cycle will occur, that is, the steeper will be the ascending branch of the curve. This indicates that the epoch of maximum does not occupy a fixed position in the cycle, and may shift in either direction depending on the cycle intensity.

Rubashev (1958) used the smoothed yearly average sunspot latitudes obtained by Gnevyshev and Gnevysheva in order to show that the latitudes of the epoch of minimum seem to be independent of the maximum Wolf number W_M. If we recall Waldmeier's conclusion concerning the high correlation between the latitude at which the cycle begins and the maximum Wolf number, then it is evident that the height W_M of the 11-year cycle is a function of the arc traversed by the sunspot zone during the cycle. It is also very significant that the average drift rate for the sunspot zone is greater the higher is the cycle intensity.

TABLE 5

Smoothed yearly average heliographic latitudes of sunspots between 1856 and 1943

Year	φ	Year	φ	Year	φ	Year	φ	Year	φ	Year	φ
Cycle 10		1870	18°.2	1888	8°.0	1904	16°.6	1921	9°.1	1937	16°.9
1856	30°.2	1871	16.0	Cycle 13		1905	14.6	1922	8.2	1938	14.9
1857	25.0	1872	14.0	1889	30.4	1906	12.9	Cycle 16		1939	13.1
1858	20.9	1873	12.4	1890	25.5	1907	11.5	1923	27.8	1940	11.6
1859	18.3	1874	11.1	1891	21.2	1908	10.3	1924	22.7	1941	10.4
1860	16.0	1875	10.0	1892	18.5	1909	9.3	1925	19.5	1942	9.4
1861	14.0	1876	9.0	1893	16.2	1910	8.4	1926	17.1	1943	8.4
1862	12.4	Cycle 12		1894	14.2	1911	7.4	1927	15.0		
1863	11.2	1879	24.0	1895	12.6	Cycle 15		1928	13.2		
1864	10.0	1880	20.3	1896	11.2	1913	22.5	1929	11.8		
1865	9.0	1881	17.8	1897	10.1	1914	21.2	1930	10.6		
1866	8.2	1882	15.6	1898	9.1	1915	18.5	1931	9.5		
1867	7.2	1883	13.7	1899	8.2	1916	16.2	1932	8.5		
Cycle 11		1884	12.2	Cycle 14		1917	14.2	Cycle 17			
1867	30.2	1885	10.9	1901	26.2	1918	12.6	1934	27.3		
1868	25.0	1886	9.8	1902	22.0	1919	11.2	1935	22.3		
1869	20.9	1887	8.8	1903	19.0	1920	10.1	1936	19.2		

In the preceding section, while discussing Spörer's law, we devoted a fairly large amount of space to a description of the second (high-latitude) sunspot zone, which appears to behave according to a reversed Spörer's law. In relation to this, it would be interesting to consider the relation between the variation in the latitude of this zone and the cyclic variation of Wolf numbers. Becker (1959), and then Kopecký (1960), have established that during cycles of increased intensity the second, high-latitude, zone reaches its highest latitudes. Consequently, there is a definite analogy between the correlations for φ_M and W_M in the two sunspot zones.

§ 5. Some Properties of the Development of Sunspot Groups

We will not attempt to consider here all known special features of the development of sunspot groups, especially properties related to the origin of new groups. Since the main subject of this study is the long-range forecasting of solar activity, therefore the properties of long-lived, or "recurrent," spot groups will be of greatest interest. It should be noted that the Greenwich term "recurrents," which is applied to these spot groups, is actually somewhat inaccurate, since they continue to exist for a long time, and do not simply recur over several solar rotations. Long-lived spot groups will here refer only to groups which exist for more than one solar rotation, and all other spot groups will be called short-lived groups.

The most important factor, from our point of view, is the lifetime of the sunspots, and the dependence of this lifetime on the other indexes describing sunspot activity. The early investigations of Maunder (1890), based on the rather meager data for the years 1886 through 1889, already established that the most frequently occurring sunspots had a lifetime of one day. Gnevyshev (1938) studied the lifetimes of spot groups on the basis of the Greenwich data for 1912—1934, which included some 3000 groups. He found that the frequency of occurrence decreased rapidly for spots of greater lifetime, as shown by Figure 5. In the same article it was shown that the spot lifetime is greater in lower heliographic latitudes.

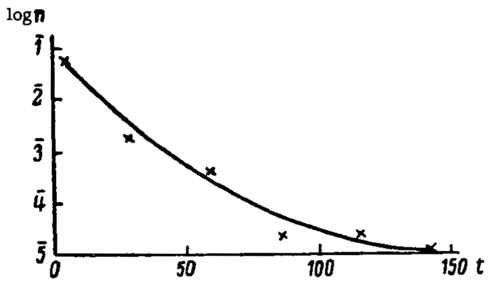

FIGURE 5

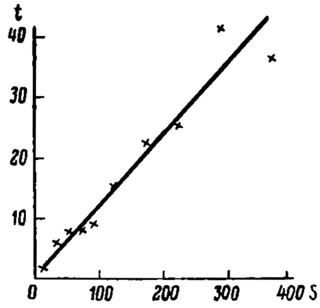

FIGURE 6

Eigenson studied the variation of the lifetimes of sunspot groups as a function of the solar-cycle phase (Eigenson, 1940; Eigenson and Prokof'eva, 1950). In this study, one solar rotation was taken as the unit of lifetime. The results showed that the ratio of the number of spot groups with lifetimes of two or more solar rotations to the number of spot groups of unit lifetime varies during the cycle.

Gnevyshev (1938) also studied the dependence of the spot-group lifetime on the group maximum area, the lifetime unit being 24 hours. He considered groups with maximum areas up to 400 millionths of a solar hemisphere and with lifetimes up to 40 days. Gnevyshev's results are plotted in Figure 6, from which it is evident that there is a linear relation between the group area and the group lifetime.

The dependence of the lifetime of a long-lived spot group (expressed in days) on the maximum area of the group was considered by Vitinskii (1958b). In this study, the Greenwich data for 1879—1950 were used, and only spot groups which appeared and disappeared at distances less than 66° longitude from the central meridian were investigated. A total of 327 long-lived groups were analyzed, and it was found that for these groups there was practically no correlation (of the type suitable for forecasting) between the group lifetime and the group maximum area:

$$r_{S_M, T} = +0.42 \pm 0.04.$$

An analysis of the extensive data available for short-lived spot groups during these same years showed that there is an essential difference, in this respect, between long-lived and short-lived groups (for the latter, $r_{S_M,T} = +0.58$). Moreover, the relation between the lifetime and maximum area for short-lived groups is, strictly speaking, nonlinear. For long-lived groups, on the other hand, it is much closer to a linear relation, a fact which, incidentally, is also indicated by the scatter of points in Figure 6 (taken from Gnevyshev's article).

For long-lived spot groups the index of maximum magnetic intensity for the group is much more significant than the maximum group area. Vitinskii (1957) applied the method of qualitative correlation in order to demonstrate that long-lived groups with maximum magnetic intensities of 3000 gauss and above have an average lifetime of three or more solar rotations. Spot groups with maximum magnetic intensities of less than 3000 gauss, on the other hand, last for no more than two rotations. Gimmel'farb (1950) has shown that the maximum magnetic-field intensity of regular spots is attained later than the maximum area. This is also the case for long-lived spot groups.

The development of spot groups in time has been considered by many authors, so that here it is enough to give only the main results, without going into detail. According to Waldmeier (1955), spot groups can be divided into structural classes which reflect both the relative locations of the spots and their sizes and shapes. The Zurich classification of Brunner and Waldmeier contains nine classes, ranging from pores through complex polynuclear groups to single regular spots. This classification is a static one, however, in that it characterizes the group structure on a given day only. No satisfactory dynamic classification of spot groups has been developed so far, since the structural evolution of spot groups can take place in very diverse ways. The following development chains, in terms of class letters, are the most common: A—B—A, A—B—C—B—A, A—B—C—D—C—H—J—A, A—B—C—D—E—F—G—H—J—A. The last two sequences are the most typical ones for long-lived spot groups.

The Brunner-Waldmeier classification is closely related to the number of spots in a group. This has led some authors (Djurcovič, 1953) to use the group-class data to prescribe a definite weight to each spot-group class, a weight which is essentially equivalent to the number of spots in the group.

The variation of the number of spots in a group during group development has been considered by Becker and Kiepenheuer (1953). The development curves for this index were found to be very nonuniform, even after reducing the parameter to the center. Since the Wolf numbers are generally determined without making such a reduction to the center, these curves have practically no forecasting value. This is also the case for curves representing the variations of the group classes.

We have still not ascertained which parameter describes group development better, the spot number or the total area. However, the curves for the area variation are more regular, and thus they can be used to some extent to predict the future development of the group. This problem has been considered in most detail for individual spots by Dyson (1925) and for entire spot groups by Ol' (Eigenson et al., 1948). A study of their area-development curves shows that the maximum area is attained on the 7th to 10th day after the appearance of the group (on the average), after which, for long-lived groups, the area gradually decreases. The area-development curves are characterized by a steep ascent and a rather gently sloping drop, and in this respect they are very similar to the cyclic curves.

§ 6. Active Longitudes

The active longitudes on the sun have a certain importance for solar-activity forecasting, and so a few aspects of this subject will now be discussed. Since a great deal of controversy has been caused by the lack of an exact definition of an "active longitude," as opposed to an "active region," therefore let us first try to define these concepts more clearly.

The active longitude is a longitudinal interval on the sun within which, for a long time (several years or more), the activity has been considerably greater than that in any other longitudinal interval.

An active region (or center of activity) is a region (or field) on the sun in which certain forms of solar activity have continuously predominated for a long time, in comparison with neighboring regions. The lifetime of an active region ranges from one to ten or more solar rotations.

These definitions indicate two essential differences between active longitudes and active regions:

1) the lifetimes of active longitudes are considerably higher than those of active regions;

2) in active longitudes, as opposed to active regions, the solar activity need not predominate continually over the activity in neighboring intervals.

Consequently, the concept of an active longitude is more comprehensive than the concept of an active region. Active regions which occur in active longitudes are more stable, particularly with respect to sunspot activity.

The following conclusions can be drawn on the basis of the many studies of active longitudes which have been made, especially those of Losh (1938), Ivanov (1933, 1935, 1936), and Vitinskii (1958a, 1960):

1. active longitudes remain in virtually the same longitudinal intervals for 2 or 3 cycles;
2. in active longitudes Faye's law of differential rotation is not valid, and the rotation is apparently rigid;
3. active longitudes are "pcpulated" mostly by long-lived spot groups with maximum areas in excess of 500 millionths of the solar hemisphere;
4. according to Losh, active longitudes in the northern and southern solar hemispheres are located at distances of approximately 180° from each other.

The objection of Becker (1955) that active longitudes do not actually exist is largely based on his subjective approach to the concept of an "active longitude." The construction of isolines for the various solar indexes has shown that, in any case, the first three conclusions are corroborated by the data for seven solar cycles. The study of active longitudes and of the active regions originating in these longitudes may throw much light on the very complicated and confusing problem of solar-activity fluctuations.

§ 7. The 22-Year Sunspot Cycle

Studies of the magnetic fields of sunspots, which were begun by Hale and his co-workers (Hale, 1913) during the 14th 11-year solar cycle, have established the existence of a 22-year sunspot cycle. Still earlier assumptions concerning the existence of such a cycle had been advanced by Wolf (Korteweg, 1883), Turner (1913b), and Ludendorff (1931). However, Hale's conclusions, contrary to those of the other investigators, were based on a very significant observational fact, namely the reversal of the magnetic polarity of sunspots from one 11-year cycle to the next. Subsequent observations of sunspot magnetic fields have confirmed this result. It has been found that during odd 11-year cycles (according to the Zurich enumeration) the polarities of the preceding [p] spots in the groups are the same as the polarity of the solar hemisphere. This regularity has so far been observed over three 22-year cycles.

Observations of the magnetic fields of sunspots have made it possible to establish the beginnings and ends of the 11-year cycles with higher accuracy. The spots of a new cycle generally start forming even before the end of the old cycle, and this overlapping of successive 11-year cycles has in some cases been more than 2 years. The use of data on the magnetic fields of sunspots has given the differentiation between successive 11-year cycles a sound physical basis, since spots may be assigned to the old or new cycle depending on their polarity.

Since studies of sunspot magnetic fields cover only a relatively short period of time (less than 60 years), it is very important to consider the problem of the stability of the 22-year cycle and to determine its relationship to the 11-year cycle. Such an analysis was first performed by Gnevyshev and Ol' (1948), who used the yearly relative spot numbers for the years from 1700 through 1944 in order to arrive at the following important conclusions.

1. The existence of a 22-year cycle is verified by all the available data, except for the data of one cycle pair (cycles 4 and 5 in the Zurich system).

2. The 22-year cycle starts with an even 11-year cycle. If we take the sums ΣW of the yearly Wolf numbers for each 11-year cycle, then we obtain the following correlation coefficients for successive 11-year cycles:

$$r_{\text{odd, even}} = +0.50 \pm 0.24,$$
$$r_{\text{even, odd}} = +0.91 \pm 0.106.$$

In subsequent studies of the 22-year spot cycle Kopecký (1950a) established that there is a quite close correlation between the respective phases of even and odd 11-year cycles. A particularly high correlation is observed for the second through fifth years after the beginning of the 11-year cycle. Kopecký also obtained very high coefficients for the correlation between the maximum yearly relative spot numbers for the even and odd cycles. For 17 Zurich 11-year cycles (with the exception of the cycle pair 4—5), the correlation coefficient was found to be $+0.765 \pm 0.106$.

Using the data for the years 1700 through 1954, Vitinskii has shown that the maximum yearly Wolf numbers for the even and odd 11-year cycles are related by the regression equation

$$W_{\text{odd}} = 0.94 W_{\text{even}} + 32.4 \, (r = +0.844). \qquad (1.19)$$

It is also an interesting fact that, according to the table of Schove (1955), the rule of Gnevyshev and Ol' is valid for 67 % of all the cases. As the studies of Gnevyshev and Ol' imply, and as equation (1.19) verifies, the height of the odd cycle is greater than the height of the even cycle. This conclusion also follows from Table 6, which is taken from the article of Kopecky (1950b).

TABLE 6

Relation between even and odd 11-year cycles (according to Kopecky)

Cycle \ Phase	0	1	2	3	4	5	6	7	8	9	10
Even . . .	8.2	31.93	59.79	77.76	75.77	68.70	52.30	33.80	24.10	12.50	8.20
Odd . . .	8.1	32.11	64.89	104.57	97.04	80.19	59.30	45.20	32.20	17.60	12.60
Ratio k . .	—	1.00	1.10	1.30	1.30	1.20	1.10	1.40	1.30	1.40	1.50

The first two rows of the table give the average yearly Wolf numbers for respective phases of even and odd 11-year cycles. The third row shows the ratios between the numbers in the second row and the numbers in the first row.

Equally important problems are those of the correlation between the various characteristics of the 22-year cycle and the correlation between the characteristics of the 22-year cycle and the two 11-year cycles which make it up. The studies of Chernosky (1954) led to the following conclusions:

1. the average Wolf number for a 22-year cycle and the sum of the relative spot numbers for this cycle are inversely proportional to the cycle duration;

2. the average Wolf number for a 22-year cycle and the sum of the relative spot numbers for this cycle are directly proportional to the duration of the next 22-year cycle;

3. the average and maximum Wolf numbers for a given 11-year cycle, and also the sum of the relative spot numbers for this cycle, are on the whole inversely proportional to the duration of the preceding 11-year cycle.

Since Chernosky's correlation coefficients are relatively low and are of no use for forecasting, we will not give his regression equations here.

A similar study was undertaken later by Chistyakov (1959), who verified Chernosky's first conclusion. Moreover, Chistyakov showed that the higher the intensity of the preceding 22-year cycle, the lower will be the intensity of the following cycle and thus the longer it will last. Finally, Chistyakov's studies also show that the correlation between the various characteristics of the 22-year cycles is no longer observed during epochs of sunspot extrema in secular cycles, a fact which will be considered below.

Some authors (Bezrukova, 1951, 1957; Chistyakov, 1959) maintain that there also exists a 44-year cycle, but it is still difficult to draw any definite conclusions concerning this, since this facet of the development of solar activity has actually been observed only during the last eight 11-year cycles. Nevertheless, this postulate concerning a 44-year cycle has given some positive results for forecasting.

The lack of a sufficiently long series of magnetic-field observations for sunspots has induced some authors (see, for example, Gleissberg, 1952) to disregard, if not to deny openly, the existence of the 22-year cycle.

However, there are certain other factors which offer an indirect proof of the existence of this cycle. For example, there are indications that there may be a 22-year period of variation in the solar diameter (Cimino, 1944), while, in addition, Tuominen (1952) has discovered a 22-year cycle in the variation of the proper motions of sunspots according to latitude.

At any rate, since the law of Gnevyshev and Ol' does not hold true for the cycle pair 4—5, it is still impossible to draw any final conclusions concerning the reversal of the magnetic polarity of sunspots from one 11-year cycle to the next. In this connection, it will be of special interest to study the magnetic fields of sunspots during low-level cycles of solar activity. At present we can only say that either the rule of alternating sunspot polarity is just not observed in some cases or else the alternation of high-level odd and lower-level even 11-year cycles is not always consistent with the reversal of the magnetic characteristics.

§ 8. The 80-Year to 90-Year Sunspot Cycle

From the curve in Figure 1, which shows the variation of the relative spot number during 18 solar cycles (according to the data of the Zurich Observatory), it is evident that in addition to the 11-year cycles this index also undergoes oscillations with much longer periods. This was first pointed out by Wolf, and subsequently various authors have studied this problem (Gleissberg, 1945; Eigenson, 1947). The main conclusion to be drawn from all these studies is that a long sunspot cycle with an average duration of 80 to 90 years also exists. The 80-year to 90-year cycle is sometimes called a "secular" cycle, because its duration is close to one century. The average duration of the 80-year to 90-year cycle and the stability of this cycle cannot yet be determined, because of the lack of relevant data. Therefore, here we will just give some results which have a direct bearing on the prediction of solar activity.

First, the 80-year to 90-year cycle shows up most clearly in the variation of the sums ΣW of the yearly Wolf numbers over the 22-year cycle, in the variation of the maximum monthly Wolf numbers W_M, and in the variation of the duration t of the 11-year cycles. Table 7 lists all these data;

TABLE 7

Main characteristics of the 80-year to 90-year sunspot cycle

Cycle No.	ΣW	W_M	t	Cycle No.	ΣW	W_M	t
0		92.6	10.2	10	1169	97.9	11.2
1		86.5	11.3	11		140.5	11.7
2	1160	115.8	9.0	12	846	74.6	10.7
3		158.5	9.2	13		87.9	12.1
4	1132	141.2	13.6	14	815	64.2	11.9
5		49.2	12.3	15		105.4	10.0
6	624	48.7	12.7	16	1016	78.1	10.2
7		71.7	10.6	17		119.2	10.4
8	1349	146.9	9.6	18		151.8	10.1
9		131.6	12.5				

columns 3 and 4 give the data of Gleissberg (1952) and column 2 gives the data of Eigenson (Eigenson et al., 1948).

If the data in columns 3 and 4 are smoothed using the formula

$$a_i = \frac{1}{2}\left(\frac{a_{i-2}+a_{i-1}+a_i+a_{i+1}}{4}+\frac{a_{i-1}+a_i+a_{i+1}+a_{i+2}}{4}\right) \tag{1.20}$$

where a_i is the index in question during the ith cycle, then we obtain the following epochs of sunspot minimum and maximum for the 80-year to 90-year cycles (according to the Zurich enumeration of the 11-year cycles):

Minimum	Maximum
6th	3rd
14th	9th

Consequently, the present cycle appears to fall in an epoch of maximum for the 80-year to 90-year sunspot cycle.

In his "eruption" hypothesis, Waldmeier (1935) involuntarily rejected all possibility of a correlation between successive 11-year cycles and also of the existence of longer solar cycles (such as the 80-year to 90-year cycle). In relation to this, it would be very interesting to consider the

TABLE 8

Coefficients of Stewart-Panofsky formula for 18 solar cycles (least-squares method)

Cycle No.	a	$a^{(4)}$	b	$b^{(4)}$	log F	[log F]$^{(4)}$	$W_M^{(4)}$
1	2.39		0.55		+0.906		
2	2.80		0.90		1.897		
3	3.75	3.09	1.11	0.85	1.792	+1.319	120.8
4	2.68	3.46	0.67	0.88	1.672	1.244	107.8
5	3.85	3.57	0.89	0.81	0.926	1.076	88.6
6	4.31	3.69	0.81	0.80	0.272	0.962	78.4
7	3.15	3.81	0.69	0.84	1.075	0.980	89.8
8	4.20	3.52	1.07	0.80	1.476	1.216	105.8
9	3.29	3.26	0.75	0.81	1.365	1.490	120.6
10	2.57	3.16	0.69	0.82	1.627	1.611	120.2
11	2.77	3.17	0.88	0.80	1.912	1.610	105.6
12	3.80	3.46	0.92	0.84	1.611	1.520	96.0
13	3.76	3.83	0.90	0.90	1.231	1.382	87.4
14	4.45	3.90	1.07	0.90	1.048	1.308	83.4
15	3.87	3.82	1.01	0.90	1.388	1.295	87.8
16	3.19	3.73	0.90	0.90	1.544	1.398	102.6
17	3.82		0.88		1.269		
18	3.66		1.10		1.917		

variation of the coefficients in the Stewart-Panofsky formula (1.10) from cycle to cycle. We observed previously that the assumption that coefficients a and b are constant constitutes only a very rough approximation. Moreover, Stewart and Panofsky (1938), using different methods, obtained sharply differing values of a, b, and F. Finally, Vitinskii considered formula (1.10) in logarithmic form and solved a system of such equations for each 11-year cycle, using the least-squares method, to obtain the values of coefficients a, b, and log F shown in Table 8.

It is evident from Table 8 that coefficients a and F exhibit a quite distinct secular variation. Smoothing according to formula (1.20) (see columns 3, 5, 7, and 8 of Table 8) gave virtually the same epochs of sunspot extrema for the 80-year to 90-year cycle as did the smoothed values of of $\Sigma W^{(4)}$ and $t^{(4)}$. Moreover, the data in Table 8 show that the variation of $a^{(4)}$ is opposite to that of $(\log F)^{(4)}$ and $W_M^{(4)}$. This is perfectly natural, since, according to equations (1.1) and (1.2), the quantities $\log W_M$ and T (the length of the rising part of the curve) vary in an opposite manner. On the other hand, coefficient b, which almost does not change, shows no secular variation and thus correlates very weakly with coefficient a.

Therefore, with all due respect to Waldmeier's "eruption" hypothesis, it is impossible to disregard completely the existence of certain correlations between the 11-year cycles. At any rate, the study of long-term solar cycles represents one of the most vital problems related to the forecasting of solar activity.

Becker (1954) showed that the rate of drift of a sunspot zone is also influenced by the 80-year to 90-year cycle. According to Gleissberg (1955), the duration of the 11-year cycle can be determined more accurately from the average rate of drift of this zone than from the cycle height. In this respect, Becker's conclusion appears to be particularly significant.

§ 9. The Asymmetry of Sunspot Activity in the Northern and Southern Solar Hemispheres

The study of the asymmetry of sunspot activity in the northern and southern solar hemispheres is closely related to the study of the properties of the 80-year to 90-year cycle. Therefore, although this factor is also allied to other aspects of solar activity, we will consider it in the present context.

First, let us observe that the 11-year solar-activity cycle constitutes a single process encompassing the entire sun. This statement is confirmed by the results of Gorbatskii (Eigenson et al., 1948) and Brunner-Hagger (Brunner-Hagger and Liepert, 1941). According to these authors, there is a quite close correlation between the spot areas in the northern and southern solar hemispheres. Using the Greenwich data for the years 1879 through 1954, we obtain the following coefficients for correlation between the maximum yearly spot areas and the sums of the yearly areas over the 11-year cycle, for the northern and southern solar hemisphere:

$$r_{S_N, S_S} = +0.63 \text{ and } r_{\Sigma S_N, \Sigma S_S} = +0.86.$$

Moreover, the correlation established by Waldmeier between the length of the rising branch of the curve and the intensity of the cycle is equally evident no matter whether the spot-area indexes for the entire solar disk or for the individual hemispheres are used (Brunner-Hagger's correlation coefficients are −0.45 and −0.44, respectively). Thus, all the features of solar-activity asymmetry in the northern and southern hemispheres represent effects of second order. It should be noted, by the way, that insofar as the Wolf numbers have not been determined separately for the two hemispheres (except for 10 years of observations at Zurich) all the studies of this problem have been based on the Greenwich data for spot areas.

A study made by Maunder (1904) of the variation of sunspot areas in the northern and southern hemispheres for the years 1874 to 1902 led to the following conclusions.

1. The spot-area curves for the 12th and 13th cycles have double maxima for the northern hemisphere and single maxima for the southern hemisphere, the single maxima being the higher of the two.

2. The single maxima coincide with the maxima for the entire sun; the points on the double maxima are separated by about three years, the trough in between corresponding to the epoch of maximum for the single-maximum cycle.

3. The epochs of extrema for cycles of double maxima occur earlier than the corresponding epochs for cycles of single maxima.

4. The cyclic curves representing the changes in latitude in the northern and southern solar hemispheres are also, respectively, single-maximum and double-maximum curves [sic].

Subsequently, these conclusions were verified, on the basis of extensive data, by Bezrukova (1951, 1957), whose studies also led to a more detailed description of the cyclic curves for the northern and southern solar hemispheres.

A study of seven 11-year sunspot cycles has shown that there are two types of 11-year cycles (in the respective hemispheres), namely single-maximum and deformed. In later works Bezrukova (1958) refers to deformed cycles as double-maximum cycles, but the application of this term to all the data studied by Bezrukova is, in our opinion, not quite justified. The deformation of a cycle expresses itself as a brief decrease in spot area near the epoch of maximum, but does not necessarily correspond to a double maximum. If in one hemisphere the cycle has a single maximum, then in the other it is deformed. Between 1878 and 1923, the forms of the cyclic curves for the northern and southern solar hemispheres alternated after every two 11-year cycles, whereas since 1924 they alternated every cycle. It should be noted that this change in the alternation pattern for the curve forms occurred in the 15th cycle, that is, near the epoch of sunspot extremum of the 80-year to 90-year solar cycle.

The alternation of low and high 11-year cycles in a Hale pair, which was established by Gnevyshev and Ol', is equally typical of both the northern and southern hemispheres of the sun. Moreover, the variation in the height of the 11-year cycle and the change in the alternation pattern of the cyclic-curve forms may indicate the existence of a 44-year cycle, a possibility which was mentioned briefly earlier.

The rising part of the curve for the 11-year cycle is mainly due to the development of the activity of a deformed cycle. This also applies to the descending part, particularly a year or two after the epoch of sunspot maximum. The maximum of an 11-year cycle is mainly determined by the maximum of the single-maximum cycle. An analysis of the data indicates a correlation between the spot areas of the secondary maximum, the preceding main maximum, or the first main maximum of the preceding deformed cycle, on the one hand, and the spot area of the maximum for the following single-maximum cycle, on the other.

It should be noted that Bezrukova used data from both the Greenwich and the Pulkovo catalogs. For this reason, her cyclic curve for the 17th cycle, in particular, does not correspond to the Greenwich data. Figure 7 gives the cyclic curves for cycles 12 through 18 for the northern and southern hemispheres (the solid curve refers to the northern hemisphere and the dashed curve to the southern hemisphere). As shown by the figure, the cyclic curves may be either single-maximum or double-maximum. Up

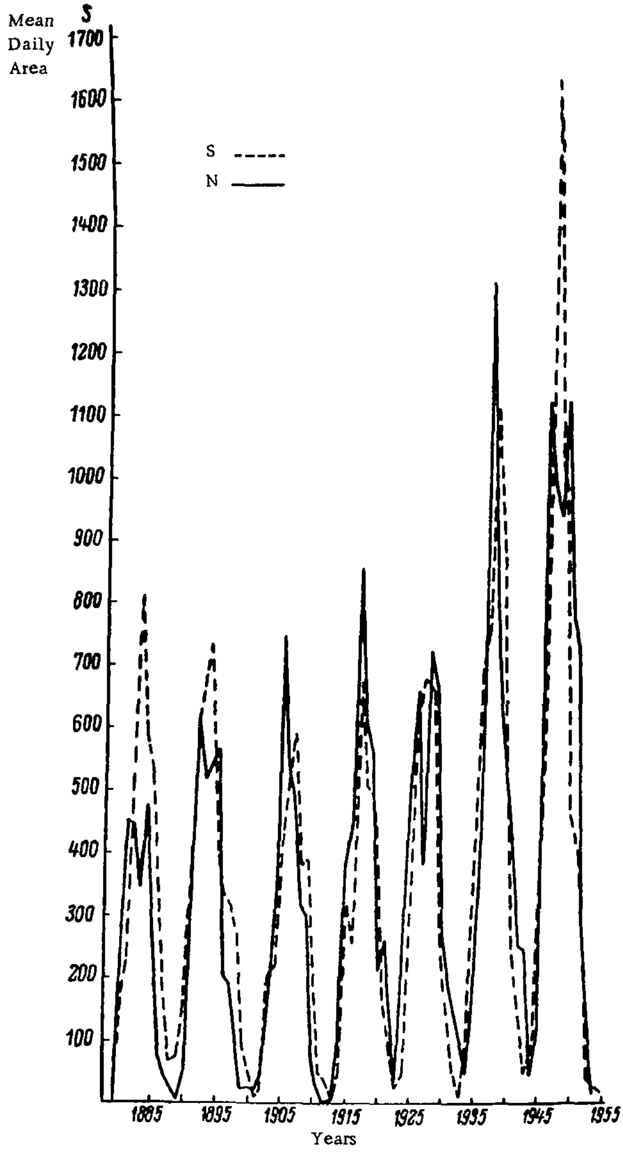

FIGURE 7

to the 14th cycle, Maunder's rule was observed. In the 14th cycle, however, a double-maximum cycle was observed for the southern hemisphere while the cycle for the northern hemisphere was single-maximum, after which the alternation of double-maximum and single-maximum cyclic curves is observed in the northern hemisphere only. In the 15th and 17th 11-year cycles the curves for both the northern and southern hemispheres were single-maximum curves.

As observed previously, one of the effects of the asymmetry of solar activity in the two hemispheres is the asynchronous development of spots, a phenomenon which is particularly pronounced near the epoch of maximum of the 11-year cycle. According to Brunner-Hagger (Brunner-Hagger and Liepert, 1941), the divergence of the epochs of maximum in the northern and southern hemispheres for cycles 11 through 17 amounts to 1.7 years, which is equivalent to an effective area decrement of 200 millionths of the solar hemisphere.

The studies of many authors (Brunner-Hagger and Liepert, 1941; Dizer, 1956; Bezrukova, 1957) have shown that the asymmetry of solar activity in the two hemispheres varies with the 80-year to 90-year cycle. To illustrate this feature, the following data (taken from Dizer's article) show the variation of the parameter $A=\frac{n-s}{n+s}$, where n and s are respectively the average spot latitudes in the northern and southern hemispheres:

1856 — 1866	1867 — 1878	1879 — 1889	1890 — 1901	1902 — 1913
+ 0.50	+ 0.11	−0.36	−0.34	−0.23

1914 — 1923	1924 — 1933	1934 — 1943	1944 — 1954
−0.15	+ 0.10	+ 0.12	+ 0.20

Since the series contains 9 members and 1 minimum, the probability of a cyclic variation is, according to the criterion of Gleissberg (1946), better than 99.5%.

Finally, it should be mentioned that all attempts to establish any periodic regularity of the distributions of spot-area differences in the northern and southern solar hemispheres within the 11-year cycle have so far failed (Vsekhsvyatskii, 1950; Bogorodskii and Zemanek, 1950).

§ 10. Fluctuations in Solar Activity

If we plot a development curve for the 11-year cycle, using the monthly values of any sunspot-activity index, then we obtain a curve which is far from smooth and which contains many secondary maxima of different intensities. These deviations from the average cyclic curve, the latter being smoothed according to some formula, are usually called fluctuations in the solar activity.

The basic feature of these fluctuations is that they outline quite clearly the 80-year to 90-year cycle (Balli, 1955). The fact that the secondary maxima and minima of the Wolf numbers and spot areas coincide in phase for 11-year cycles with rising parts of the curve which are approximately equal in length is considered to be an indirect proof of this correlation (Xanthakis, 1957; Xanthakis, 1959). The fluctuations of the relative spot numbers show no periodicity within the 11-year cycles and so can essentially be considered as independent processes (Vitinskii, 1961b). The properties of the solar fluctuations (their amplitude and duration only, their intensity being disregarded here) do not show essential differences over the rising and descending parts of the 11-year cycle. This also applies to the spot areas, since a very close correlation exists between the Wolf numbers and the sunspot areas.

Strong fluctuations in solar activity are the most important for forecasting purposes, since they constitute the main source of errors in practically all empiricostatistical methods for the forecasting of solar indexes. Strong fluctuations are here understood to mean deviations from the mean cyclic curve which are in excess of one standard deviation. An analysis of the data listed in the catalog of strong fluctuations for the years 1755—1954 (Vitinskii, 1960b, 1960c) leads to the following conclusions.

1. Practically all the cycles show strong fluctuations in Wolf numbers during the first year prior to the epoch of maximum of the solar cycle and during the three years after this epoch. An analogous conclusion was reached by Bezrukova (1958), as a result of an analysis of spot areas in the northern and southern solar hemispheres.

2. The higher the intensity of the 11-year solar cycle, the lower will be the density of the strong fluctuations (the fewer fluctuations will be observed per annum in the rising part of the curve). Low-level cycles, conversely, have a quite high density of fluctuations during their rise periods.

3. There is a fairly sharp distinction between the properties of the strong fluctuations during the rising and descending parts of the 11-year cycle. During the period of rise, the density of strong fluctuations is primarily determined by the duration of the rise, whereas during the period of descent it is determined both by the duration of the descent and by the maximum Wolf number.

Incidentally, attempts which have been made to determine the yearly variation of Wolf numbers (see, for example, Vsekhsvyatskii, 1950) can hardly be considered successful. Loewe and Radok (1959) have shown that the yearly wave of monthly relative spot numbers is not stable and should rather be considered as one of the random fluctuations in a strictly autocorrelated time series.

A detailed study of the fluctuations of sunspot areas in the northern and southern solar hemispheres was made by Bezrukova (1958). Her results indicate that the maximum area fluctuation in a single-maximum cycle occurs at the 16th fluctuation, where the fluctuations are numbered from the beginning of the 11-year cycle regardless of their intensity. The maximum fluctuations of deformed cycles are on the average the 5th, 8th, and 11th fluctuations in the rise period and the 15th, 19th, 22nd, and 29th in the descent period. Thus, in this case a gap of 3 or 4 fluctuations is observed between successive high fluctuations. The highest fluctuations in a deformed cycle are the 9th and 24th.

§ 11. Some Remarks Concerning Long-Duration Solar Cycles

Although the reliable data which are available at present on the Wolf numbers cover over nineteen 11-year sunspot cycles, these are still insufficient for reaching any definite conclusions concerning the possible existence of long-duration solar cycles. It is true that the table of Schove (1955) contains qualitative data on sunspot activity which goes back to the year 649 B.C., obtained from descriptions of the polar auroras, but these data can only with a certain reservation be utilized for estimating long-period solar-activity cycles.

However, some quantitative estimates can be made without using Schove's data. Gleissberg (1944a), in an analysis of the ratio Q of the lengths of the rising and descending parts of the 11-year cycle, observed a systematic decrease in this parameter during the years between 1615 and 1937. Consequently, he concluded that there may exist a sunspot cycle with a duration of some thousand years. Rubashev (1949) discovered a 600-year cycle on the basis of the variation in the number of comets which were visible to the naked eye, and finally Maksimov (1953) established the existence of a cycle of like duration from the variation in the thickness of tree rings.

If we now consider the data in Schove's table, we see that at the end of the 17th century solar activity reached an all-time low for the period of telescopic solar observations, whereas a cycle comparable in intensity to the present cycle last occurred from 1368 to 1378 (about 600 years ago). Therefore, there is some indication that a solar-activity cycle about 600 years in length exists.

It was mentioned previously that so far we cannot hope to determine the duration of the 80-year to 90-year cycle with any real accuracy, and that this duration is apparently subject to very strong fluctuations. For example, Schove's table indicates that this cycle can be anywhere from five to eleven 11-year cycles in length. Of course, these results only represent estimates, but they suffice to show that cycles with durations of fifteen (Anderson, 1954; Djurcovič, 1956) or sixteen (Bonov, 1957) 11-year cycles are extremely unlikely. Actually, for the first case to be true, two adjacent secular cycles would have to have durations of seven and eight 11-year cycles, whereas in the latter case the adjacent cycles would both be eight 11-year cycles in duration. It is significant that in his most recent article Bonov (1961) refers only to a long sunspot cycle, without mentioning its duration explicitly. Therefore, the problem of long-duration solar-activity cycles has so far only been posed, and we are still very far from a solution.

§ 12. Concluding Remarks

In conclusion let us stress once more that in the present chapter we have not attempted to give a detailed representation of all the fundamental properties of solar activity. Rather we have only discussed certain problems which have a direct bearing on the forecasting of sunspot-activity indexes.

In relation to this, we have not been concerned with solar activity in the chromosphere and corona. Moreover, for the same reason, some very basic problems of sunspot activity have been disregarded entirely, whereas certain phenomena which, though of secondary significance for the overall study of solar activity, are nevertheless important with respect to solar forecasting have been considered in great detail.

Chapter II

SOLAR-ACTIVITY FORECASTS A YEAR OR SEVERAL YEARS IN ADVANCE

§ 1. Introductory Remarks

In this chapter we will consider basic methods for the long-range forecasting of Wolf numbers a year or several years in advance, within the limits of the current cycle. These methods are the most reliable of all the empiricostatistical forecasting methods available at present. In addition, they are quite accurate, mostly due to the diversity of the methods, the errors of which mutually compensate one another.

As observed previously, the ascending and descending parts of the 11-year sunspot cycle have essentially different characteristics, so that it is advisable to consider separately methods for forecasting the Wolf numbers for the two branches. It should also be noted that some authors compute, instead of the yearly value, the smoothed monthly Wolf numbers for the corresponding separate epochs of the cycle. Thus, in each individual case it will be specified which relative spot numbers are actually being considered.

The "eruption" hypothesis of Waldmeier (1935) has had special significance for the development of methods for forecasting Wolf numbers within the current cycle. Before the advent of this hypothesis, investigators were mainly concerned with studying real, but more often imaginary, periodicities in solar activity in such a way that a great number of mostly unsuccessful ultralong-range forecasts resulted. The "eruption" hypothesis, however, forced researchers to concentrate exclusively on the basic inner regularities of the 11-year cycle. The principal results of these studies were discussed in Chapter I, so that it will not be necessary to repeat them here. It should be noted, though, that the forecasts obtained on the basis of these methods have been the most trustworthy. Since Waldmeier was the first to offer a successful forecast of the Wolf numbers for the current cycle, let us begin by discussing his method.

§ 2. Waldmeier's Method

Waldmeier's method for forecasting solar activity within the current cycle is based on relations (1.1) through (1.6). The first two of these formulas can be replaced quite accurately by the single formula

$$\log W_M = 2.58 - 0.14T, \tag{2.1}$$

from which we obtain

$$T = 18.4 - 7.14 \log W_M. \tag{2.2}$$

Here, W_M is the smoothed monthly maximum Wolf number and T is the duration of the rising part of the cycle, in years.

It will be convenient to introduce another very important characteristic of the 11-year solar cycle, namely the rate of its development

$$V = \frac{W_M - W_m}{T}, \tag{2.3}$$

where W_m is the minimum smoothed monthly Wolf number. From (2.2), we have

$$V = \frac{W_M - W_m}{12\,(18.4 - 7.14 \log W_M)}. \tag{2.4}$$

If this parameter is determined at the beginning of the cycle, then it can be used to forecast W_M.

This method was first applied by Waldmeier to predict the height and the epoch of maximum of the 17th sunspot cycle, and it was found to be exceptionally dependable, especially for that time (Waldmeier, 1936). Waldmeier used the smoothed monthly Wolf numbers for the period from 1933 to October 1935 to obtain values of 124 for the height of maximum and 1937.7 for the epoch of maximum. The actual height of the maximum of the 17th cycle was 119.2, while the epoch of maximum was 1937.4. No less successful was Waldmeier's forecast for the 18th solar cycle (Waldmeier, 1946):

	Epoch of maximum	Height
Observed	1947.5	151.8
Forecast	1947.6	139

The above basic propositions of Waldmeier's method are of particular significance, and thus deserve special attention. In practice, Waldmeier's method also makes it possible to predict the Wolf numbers for the descending part of the cycle. In order to do this, an analog cycle is selected on the basis of the data for the beginning of the current cycle, and the descending part of this analog cycle is used to predict the Wolf numbers for the descending part of the current cycle. It should be noted, however, that the W values forecast for the descending branch using this method were somewhat low.

Ol' (1949a) investigated the reliability of Wolf numbers forecast for the ascending and descending parts of the cycle using this method. He plotted the mean cyclic curves for two groups of cycles, namely high cycles with $W_M \geqslant 100$ and low cycles with $W_M < 100$ (cycle 4 was not considered). Ol' then computed the reliability of the average values W_i^0 ($i = -5, -4, \ldots, 0, +1, \ldots, +7$) obtained in this way, with an accuracy of 10%, from the corresponding W_i^0 ($i=0$ in the epoch of maximum). For the high cycles ($W_M \geqslant 100$) the average reliability for the ascending part of the cycle was 0.51, while for the descending part it was 0.76. Consequently, for high cycles, forecasts can be made for the descending part only. For low cycles ($W_M < 100$) the average reliability for the ascending and descending branches was 0.49 and 0.53, respectively. Consequently, the average cyclic curve for low cycles is completely inadequate for forecasting. Thus, to summarize, Wolf-number forecasts for the descending part of the curve, obtained by the analog method, are unreliable even for high cycles.

§ 3. The Cyclic-Curve Method

Stewart and Panofsky developed Waldmeier's analog method and "eruption" hypothesis further to formulate a method based on their two-parameter formula (1.10)

$$W = F\theta^a e^{-b\theta}.$$

Whereas Waldmeier's method is actually based on a one-parameter family of cyclic curves, in the method of Stewart and Panofsky constants F, a and b are assumed to change from cycle to cycle (Stewart and Panofsky, 1938). Using the data for solar activity up to 1938, Stewart and Eggleston (1939) determined the constants in equation (1.10) for the 17th cycle and then predicted the smoothed monthly Wolf numbers for the descending part of this cycle. This forecast is shown as follows, the figures in parentheses being the observed values for the given months (or, more precisely, the smoothed values):

VII 1939 — 87.2(87.6),	VII 1941 —37.7(47.1),
I 1940 — 73.5(73.5),	I 1942 —28.9(43.8),
VII 1940 — 60.3(67.6),	VII 1942 — 21.8(29.6).
I 1941— 48.2(56.6),	

These data indicate that the cyclic-curve method gives values for the relative spot numbers on the descending curve which are too low. Moreover, the three parameters cannot be determined unless a quite considerable portion of the cyclic curve is available, a factor which limits the forecast severely.

In addition the cyclic-curve method was subsequently employed by Cook, who plotted a one-parameter family of cyclic curves on the assumption that parameters b and F are related to a (Cook, 1949). The parameter a is best determined from the curve for the relation between a and W_M. However, Cook's method predicts Wolf numbers with such a low accuracy that it is hardly worthwhile to use it for forecasting purposes.

Finally, Chvojkova (1952) has approximated the 11-year cyclic curves using the expression

$$W = \frac{W_M}{2}\left(1 - \cos\frac{2\pi t}{aT + (1-a)t}\right). \tag{2.5}$$

Here, t is the time from the beginning of the cycle, T is the duration of the cycle, and $a = \frac{T_1/T}{1 - T_1/T}$, where T_1 is the time from the beginning of the cycle to the epoch of maximum (that is, the length of the rising part of the curve). This equation can also be used to predict the Wolf numbers on the descending part of the cyclic curve. The parameters W_M, a and T can be selected according to the rising part of the given cycle. It should be observed, however, that Chvojkova's formula is in no way superior to the Stewart-Panofsky formula. The Wolf numbers obtained using this formula are also too low, and the results apply to a very restricted portion of the cyclic curve.

Consequently, the cyclic-curve method does not ensure an adequate reliability for forecasts of values on the descending branch. In addition, its applicability is highly restricted, so that it is only of historical significance with respect to forecasts of Wolf numbers for the current cycle. Later, however, we will show that this method is useful to some extent in ultralong-range forecasts.

§ 4. Gleissberg's Method

Waldmeier's method and the cyclic-curve method are both based on the assumption that the 11-year solar cycles are entirely independent of one another. Gleissberg (1939), on the other hand, carried out certain studies which indicated that successive cycles are mutually dependent and show a long-period variation pattern. Consequently, Gleissberg's method for forecasting the Wolf numbers is only partly a method for predicting the Wolf numbers for the current cycle. It might be classified more appropriately as a method for ultralong-range forecasting. Therefore, here only Gleissberg's article (1940), in which he first gave a forecast for the descending part of the 17th sunspot cycle, will be considered. The details of the method will be discussed in Chapter IV.

Since the epoch of minimum of the solar cycle cannot be pinpointed due to the overlapping of two successive cycles, therefore Gleissberg introduced a reduced length t_r for the rising part of the cycle and a reduced length t_f for the falling part, these being determined as follows. If W_M is the highest smoothed monthly Wolf number in the cycle, t_M is the month of occurrence of this maximum number, and t_1 and t_2 are the months in the rising and descending parts of the cycle, respectively, in which the smoothed monthly Wolf numbers are equal to $\frac{1}{4}W_M$, then t_r and t_f are defined as

$$t_r = t_M - t_1 \text{ and } t_f = t_2 - t_M. \tag{2.6}$$

Using the Zurich data, Gleissberg determined W_M, t_r, and t_f for cycles 0 through 17. Then, he took the average of four successive 11-year cycles to obtain the quantities $W_M^{(4)}$, $t_r^{(4)}$, and $t_f^{(4)}$. Table 9, in which these values are listed, shows that the long-period variation of $W_M^{(4)}$ is accompanied by analogous variations of $t_r^{(4)}$ and $t_f^{(4)}$. The maxima and minima of $t_f^{(4)}$ almost coincide with the maxima and minima of $W_M^{(4)}$, whereas the maxima of $t_r^{(4)}$ correspond to the minima of $W_M^{(4)}$ and vice versa. Moreover, the sum $t_r^{(4)} + t_f^{(4)}$, which is given in the last column of Table 9, remains practically constant. It is important to note that the data in the last five columns of the table refer to cycles 0—3, 1—4, ..., 14—17, 15—18, 16—19, respectively.

The figures in the last column of Table 9 show that the sum $t_r^{(4)}+t_f^{(4)}$ always lies somewhere between 84 and 92. Gleissberg thus assumed that for cycles 14—17 this sum would not be less than 84. Consequently, since the corresponding $t_r^{(4)}$ was 33, the next value of $t_f^{(4)}$ should not have been less than 51. This quantity, then, represents the average of t_f for cycles 14 through 17. Also, the sum of these four numbers should at any rate not be less than 203, and since the sum of the three numbers (for cycles 14, 15, and 16) in column 5 of Table 9 is 140, therefore t_f for the 17th cycle should not be less than 63 months. In the 17th cycle, $W_M = 119.2$, that is, $\frac{1}{4} W_M = 30$. Therefore, the smoothed monthly Wolf number should drop to 30 no earlier than 63 months after the epoch of maximum (April 1937), that is, not before the second half of 1942. Actually, the smoothed monthly relative spot number first dropped below 30 in July 1942. Therefore, this first application of Gleissberg's method was here completely successful and even represented some progress in comparison with Waldmeier's forecast.

TABLE 9*

Principal characteristics of 11-year sunspot cycles (according to Gleissberg)

Cycle number	W_M	t_l	t_r	t_f	$W_M^{(4)}$	$t_l^{(4)}$	$t_r^{(4)}$	$t_f^{(4)}$	$t_r^{(4)}+t_f^{(4)}$
0	92.6	—	—	41	113.4	—	—	49	—
1	86.5	41	52	48	125.5	33	31	59	90
2	115.8	22	29	58	116.2	40	33	54	87
3	158.5	29	17	48	99.4	52	34	53	87
4	141.2	41	28	81	77.7	59	43	49	92
5	49.2	66	57	27	79.1	56	42	42	84
6	48.7	73	35	54	99.7	51	36	52	88
7	71.7	57	51	35	112.0	44	35	56	91
8	146.9	29	24	52	129.2	37	29	60	89
9	131.6	45	34	68	111.2	45	34	56	90
10	97.9	45	31	70	100.2	47	35	53	88
11	140.5	30	26	51	91.8	49	36	49	85
12	74.6	62	47	34	83.0	55	38	48	86
13	87.9	51	36	57	83.9	49	37	49	86
14	64.2	53	35	54	91.7	47	33	51	81
15	105.4	52	32	46	113.6	43	30	52	82
16	78.1	39	43	40	137.6	40	29	—	—
17	119.2	45	23	63					
18	151.8	37	21	58					
19	201.3	41	31	—					

* Table 9 also gives the data for cycles 18 and 19, taken from later works of Gleissberg (Gleissberg, 1953). In addition, the values of t_l and $t_l^{(4)}$, which will be defined in Chapter 4, are also given here.

§ 5. The Latitude Method

As mentioned previously (see Chapter I, § 4), the Wolf numbers in the descending part of the 11-year solar cycle are completely determined by the heliographic latitude, this dependence being the same for all cycles. This regularity is most reliably observed for latitudes of $\varphi < 14°$. The latitude method is based on this dependence, which was established by Gnevyshev and Gnevysheva (1949). The form of the relation between the corresponding φ and t for the descending part of the sunspot cycle was shown above in Figure 4.

Let us now use the curve in Figure 4 and the yearly average latitudes for the rising part of the given 11-year solar cycle to forecast the yearly Wolf numbers for the descending part of the same cycle. In order to do this, we will plot the latitudes for the rising part and the beginning of the descending part of the curve (up to 14°) onto tracing paper and then superimpose the tracing paper onto the mean $\varphi=\varphi(t)$ curve in Figure 4 in such a way that the points on the tracing paper fit the curve best (the tracing paper is moved along the abscissa axis to find the best fit).

Using the mean curve we then transpose onto the tracing paper the continuation of this curve for the descending part of the given cycle and read off the latitudes φ at one-year intervals. These values of φ now allow us to plot the relation between φ and W for $\varphi < 14°$ found by Gnevyshev and Gnevysheva (see Figure 8). In this way we obtain the yearly relative spot numbers for the descending part of the 11-year solar cycle.

Using the latitude method, Ol' (1954) computed the yearly Wolf numbers for the descending parts of cycles 10 through 18. The [overall] standard deviation of the values obtained was 11.8.

As an example of a forecast based on the latitude method, let us predict the yearly relative sunspot numbers for the descending part of the 18th cycle:

	1949	1950	1951	1952	1953	1954
φ	13°.3	11°.9	10°.7	9°.5	8°.6	7°.9
W	86	61	43	27	17	10
W_{obs}	135	84	69	31	13	4

A comparison of the observed numbers W_{obs} with the forecast numbers W indicates that the high values of W_{obs} in the middle of the descending branch (1949—1951) were not foreseen. For the subsequent years, however, there is a satisfactory agreement between W_{obs} and W.

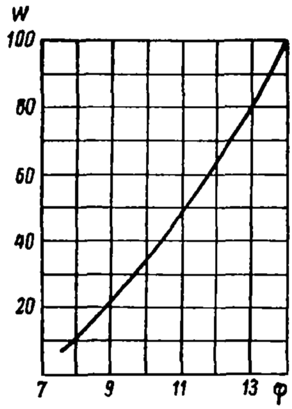

FIGURE 8

Just as in the case of Waldmeier's method, the main advantage of the latitude method is its simplicity. If it is also kept in mind that its reliability is higher than that of the analog method, then this method recommends itself as one of the working methods for predicting the descending part of the cycle. Unfortunately, however, its range is very restricted (it is applicable for $\varphi < 14°$ only). Moreover, the latitude method does not take into account fluctuations in the yearly Wolf numbers (and in the latitudes), a fact which is evident from the following forecast for the 19th cycle. The following yearly Wolf numbers were obtained for the descending part of the current (19th) cycle: 1961—64 (51), 1962—52, 1963—34, 1964—28, 1965—13.

§ 6. Kozik's Method

Kozik's method is based on the use of a new sunspot-activity index $K=10\sqrt{W}$, which was first introduced by Omshanskii (according to

Kozik, 1949). While applying this index in order to simplify the 11-year sunspot-cycle curve, Kozik (1946) discovered the following advantages of the index over the Wolf numbers:

1) the use of the index K levels off the fluctuation amplitudes for different phases of the 11-year solar cycle;

2) the scatter of heights of the minima and maxima of the 11-year cycles is of the same order of magnitude for the index K;

3) the correlations with certain geophysical indexes become more simple and linear when the index K is used.

The cyclic curves for the index K constructed by Kozik (1949) illustrate the four fundamental points characterizing an 11-year cycle, namely, the beginning of the cycle, the first break point, the second break point, and the end of the cycle. If all the 11-year solar cycles are divided into three groups according to intensity, namely strong ($W_M > 110$), average ($80 \leqslant W_M \leqslant 110$), and weak ($W_M < 80$), then we find that in strong and average cycles the second break point coincides with the epoch of sunspot maximum. For low cycles the second break point is either located on the same level as the first break point or else even lies below it. Unfortunately, the first and second break points can be determined only with low reliability, and this naturally affects the reliability of forecasts for low solar cycles.

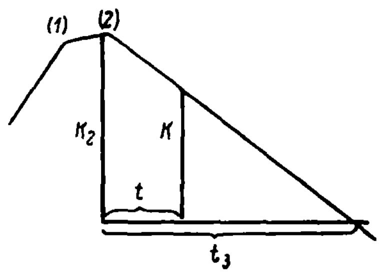

FIGURE 9

The general form of Kozik's cyclic curve is shown in Figure 9, in which the following notation is used: (1) and (2) are the first and second break points; K_2 is the value of index K at the second break point; and t_3 is the time from the second break point to the point where $K = 0$.

Although Kozik derived relations describing both the descending and rising parts of the 11-year cycle, we will consider here only his results for the descending part.

The initial assumption is that during the descending part of the cycle the index K is a linear function of time. Moreover, the rate of decrease of K in the descending part is apparently independent of the height of the given cycle and averages 11 to 13 per annum. From an analysis of 17 cycles (cycles 3 and 4 in the Zurich enumeration were excluded), Kozik found that

$$t_3 = 0.08K_2 \, (\sigma = \pm 0.59 \text{ yr}). \tag{2.7}$$

If t is the time reckoned from the second break point, then from Figure 9 it is evident that

$$\frac{K}{t_3 - t} = \frac{K_2}{t_3}, \quad \text{or} \quad K = K_2 - \frac{K_2}{t_3} t.$$

Now, from equation (2.7) we find that

$$K = K_2 - 12.5t, \tag{2.8}$$

and, since $K=10\sqrt{W}$, formula (2.8) gives the yearly values.

We should also note the following factor. Since the preceding formulas were derived from the monthly values of W, therefore K should be computed for the middle of each year and then, after converting from K to W, unity should be added to the result to obtain the yearly Wolf numbers.

Kozik predicted the following yearly Wolf numbers for the descending part of the 18th sunspot cycle:

	1948	1949	1950	1951	1952	1953	1954	1955	1956
K	104	91	79	66	54	41	29	16	4
W	109	84	63	45	30	18	9*	4*	
W_{obs}	136	135	84	69	31	13	4		

The last row gives the observed Wolf numbers W_{obs}. Asterisks indicate the numbers which are low according to Kozik; the table shows that Kozik's method mostly results in relative yearly spot numbers which are too low.

According to Ol' (1954), back calculations of the yearly Wolf numbers for the descending parts of solar cycles 0 through 18 (Zurich system) according to Kozik's method have an overall standard deviation of 10.8. It is typical that the highest standard deviations are observed for cycles whose descending parts have sharp fluctuations (cycles 9 and 18), and also for cycles for which Kozik's relation between K_2 and t_3 is not valid (cycles 3 and 4).

The main advantage of Kozik's method is its simplicity. The main disadvantages are that it does not take into account the possibility of fluctuations of Wolf numbers and that the determination of the second break points for low solar cycles is difficult.

§ 7. The Regression Method

As mentioned in § 1 of this chapter, Ol' (1949a) investigated the reliability of Wolf-number forecasts according to Waldmeier's method. To do this, Ol' used the cyclic curve for high cycles ($W_M > 100$) to obtain the following forecast for the yearly Wolf numbers in the descending part of the 18th sunspot cycle (the actually observed numbers are given in parentheses):

1948—126(136),	1951—50(69),
1949—101(135),	1952—32(31),
1950— 76(84),	1953—14(13).

These data indicate that the reliability of this forecast is no lower (especially after 1950), and in some cases is even higher, than that of Kozik's foreeast.

A further refinement of this method is the regression method developed by Ol' in 1954 (Ol', 1954). The regression method is based on the fact that within a given cycle each successive sunspot number can be correlated with the preceding one. If W_i is the yearly number for a given year and W_{i+k} is the yearly number after k years, then

$$W_{i+k}=a_kW_i+b_k, \tag{2.9}$$

where a_k and b_k are constants determined from an analysis of the relative Wolf numbers for the 19 known solar cycles (cycles 0 through 18 in the Zurich system). Naturally, only correlations for which the correlation coefficient was $r(W_i, W_{i+k})>0.60$ were suitable for forecasting purposes.

In contrast to the previous methods, the regression method makes it possible to predict Wolf numbers not only for the descending part of the given cycle but also for the rising part as well. This is very significant, since this method can thus be used at the very beginning of the cycle to estimate the development of the current cycle.

Let us now introduce the following notation: W_M and W_m are, respectively, the maximum and minimum relative sunspot numbers; W_{M+k} and W_{m+k} are the Wolf numbers k years after the corresponding epochs of extremum, where for the descending part $k=1,2,\ldots,7$ and for the rising part $k=1,2,3$.

The following relations hold true for the descending part of the solar cycle:

$$\left.\begin{array}{lll}
W_{M+1}=0.87W_M-4 & r=0.95 & \sigma=\pm 10.3\\
W_{M+2}=0.76W_M-8 & r=0.86 & \\
W_{M+3}=0.62W_M-15 & r=0.88 & \\
W_{M+4}=0.41W_M-7 & r=0.74 & \\
W_{M+2}=0.90W_{M+1}-8 & r=0.93 & \sigma=\pm 9.2\\
W_{M+3}=0.72W_{M+1}-12 & r=0.93 & \\
W_{M+4}=0.50W_{M+1}-8 & r=0.81 & \\
W_{M+5}=0.41W_{M+1}-12 & r=0.78 & \\
W_{M+6}=0.22W_{M+1}-7 & r=0.60 & \\
W_{M+3}=0.75W_{M+2}-3 & r=0.94 & \sigma=\pm 7.5\\
W_{M+4}=0.53W_{M+2}-2 & r=0.84 & \\
W_{M+5}=0.33W_{M+2} & r=0.66 & \\
W_{M+6}=0.31W_{M+2}-9 & r=0.68 & \\
W_{M+4}=0.76W_{M+3}-3 & r=0.92 & \sigma=\pm 7.1\\
W_{M+5}=0.58W_{M+3}-6 & r=0.86 & \\
W_{M+6}=0.48W_{M+3}-12 & r=0.82 & \\
W_{M+7}=0.44W_{M+3}-13 & r=0.76 & \\
W_{M+5}=0.76W_{M+4}-3 & r=0.89 & \sigma=\pm 7.8\\
W_{M+6}=0.68W_{M+4}-7 & r=0.81 & \\
W_{M+7}=0.52W_{M+4}-11 & r=0.66 & \\
W_{M+6}=0.69W_{M+5}-4 & r=0.91 & \sigma=\pm 3.5\\
W_{M+7}=0.64W_{M+5}-7 & r=0.83 & \\
W_{M+7}=0.85W_{M+6}-3 & r=0.93 & \sigma=\pm 4.1
\end{array}\right\} \tag{2.10}$$

Here, the correlation coefficients r are given in each case, and the standard deviations σ of the predicted Wolf numbers for cycles 0 through 18 are indicated for the cases with the highest correlation coefficients. The overall standard deviation for this set of W_{M+k} values is 7.8. It should be noted that the analysis which Ol' made of the applications of this method refers to the most favorable conditions, in which the values are predicted from year to year (in contrast to the latitude method and Kozik's method). Naturally, if the forecast is made a longer time in advance, especially for the entire descending part of the cycle, the results are much poorer since the correlation is weaker. The regression method does not apply to the end portion of the descending part of the cycle ($k > 4$), since here the correlation coefficients for W_M are too small.

As a sample application of this method, we now give the yearly Wolf numbers forecast for the descending part of the 19th sunspot cycle (Ol', 1960): 1961—84, 1962—58, 1963—36, 1964—20, 1965—10. The last two figures were extrapolated on the basis of the shape of the cyclic curve.

As mentioned previously, the regression method also makes it possible to predict the yearly relative spot numbers for the rising part of the cycle. In terms of the previous notation, the following equations may be obtained for the rising branch:

$$\left.\begin{aligned} W_{m+2} &= 1.953W_{m+1} + 17 \quad r = 0.83 \\ W_{m+3} &= 1.592W_{m+2} + 6 \quad r = 0.97 \\ W_{m+3} &= 3.067W_{m+1} + 33 \quad r = 0.79 \end{aligned}\right\} \qquad (2.11)$$

The correlation coefficients are indicated for each case. Here there was no point in computing W_{m+4}, since for many cycles $W_{m+3} = W_M$, so that in some cycles $W_{m+4} < W_{m+3}$ while in others $W_{m+4} > W_{m+3}$.

Back calculations for cycles 1 through 18, using formulas (2.11), gave the following standard deviations: 13.8, 11.6, and 26.1. The last regression, of course, is useless for forecasting.

The maximum Wolf number W_M for a given cycle can be computed at the beginning of the cycle using the following equations:

$$\left.\begin{aligned} W_M &= 1.233(W_{m+2} - W_m) + 49 \quad r = 0.87 \\ W_M &= 1.622(W_{m+2} - W_{m+1}) + 49 \quad r = 0.87 \end{aligned}\right\} \qquad (2.12)$$

The correlation between W_{m+1} and W_m was found to be insufficient. Back calculations for cycles 1 through 18 gave standard deviations of 16.7 and 15.8. If we take the average of the results obtained using these two formulas, then we obtain a standard deviation of 15.3.

Subsequent studies have shown that if these theoretically obtained values W'_M are subtracted from the observed values W_M, then the difference $W_M - W'_M = \Delta$ shows a regular several-year variation (from cycle to cycle). In order to investigate the regularity of this variation, sets of four values of Δ were smoothed and the curve for $\Delta^{(4)}$ was plotted. This curve can be approximated by the following sinusoidal function of time:

$$\Delta^{(4)} = -14.5 \sin(36t + 7.2), \qquad (2.13)$$

where t is the time, expressed as the number of the given 11-year sunspot cycle minus 3 (for example, for the 18th cycle $t = 15$, while for the current 19th cycle $t = 16$). Using formula (2.13), the values of $\Delta^{(4)}$ were computed for all known past cycles, and then the values of Δ were found using the formula

$$\Delta_k = 4\Delta^{(4)}_{k-3} - (\Delta_{k-3} + \Delta_{k-2} + \Delta_{k-1}).$$

The corrections Δ improved the accuracy of the forecasts, so that the standard deviations dropped from 15.3 to 9.5. Let us note that formula (2.13) can also be used to forecast the value of Δ for the next cycle (for the 19th cycle, $\Delta = 36$).

Therefore, the regression method enables a fairly accurate prediction of the rising part of the cyclic curve, but the range of the forecast is lower than that for the descending part. In practice, the rising part can be predicted a year, and in some cases two years, in advance.

To illustrate the reliability of a forecast made for the rising part of the curve using the regression method, let us give the results obtained for the rising part of the 19th sunspot cycle. It should be noted that according to various authors the maximum of the 19th sunspot cycle was expected in 1957. If 1954 is taken as the epoch of minimum, then we obtain 91 (147) for 1956 and 200 (190) for 1957. A comparison with the observed values (given in parentheses) shows that the accuracy of this forecast is satisfactory.

Finally, let us mention that the regression method, when used to forecast values from year to year, takes the Wolf-number fluctuations into account, but that it "overcompensates," since after the fluctuation the Wolf numbers predicted by this method are generally too high.

§ 8. MacNish's Method

MacNish's method (MacNish and Lincoln, 1949), which is closely related to the regression method, is based on the following assumptions:

1) in a cyclic time series (for example, the series of Wolf numbers), any future value can be estimated, to a first approximation, as the average of all the past values of W for the same phase of the sunspot cycle;

2) this estimate should be corrected in proportion to the deviations which the earlier values of W in the same cycle show with respect to the corresponding averages.

We then have

$$W'_n = \bar{W}_n + \Delta W'_n = \bar{W}_n + k_{1,n}\,\Delta W_{n-1} + k_{2,n}\Delta W_{n-2} + \ldots,$$

where n is the number of years after the beginning of the cycle, W'_n is the forecast value of W for the nth phase of the cycle, $\bar{W}_n$ is the average of all the W_n for earlier cycles, ΔW_{n-1}, ΔW_{n-2}, ... are the differences between the observed (W) and average $(\bar{W})$ values for the phases of the same cycle which precede the nth phase, and $k_{1,n}$, $k_{2,n}$, ... are forecasting coefficients determined by the least-squares method.

In practice only the coefficient $k_{1,n}$ is enough, so that the Wolf numbers may be predicted using the simpler formula

$$W'_n = \bar{W}_n + k_{1,n}\Delta W_{n-1}. \qquad (2.14)$$

MacNish obtained the following values of $k_{1,n}$ for the years after the epoch of minimum of the 11-year cycle:

1	2	3	4	5	6	7	8	9	10	11
1.2	2.4	1.4	0.6	0.5	0.7	0.4	1.1	0.8	0.7	0.3

It

It should be noted that MacNish used smoothed yearly relative sunspot numbers in his calculations.

In order to apply MacNish's method in practice, Ol' (1954) computed the coefficients $k_{1,n}$ for the unsmoothed yearly Wolf numbers. A certain simplification was achieved in the following way. Instead of using the least-squares method, the average ratios

$$\frac{(W_n)_{\mathrm{obs}} - \overline{W}_n}{(W_{n-1})_{\mathrm{obs}} - \overline{W}_{n-1}} = k_{1,n}$$

were computed. Then, a smooth curve was drawn through the values obtained for $k_{1,n}$, and the following values of k_{1n} were read from this curve:

1	2	3	4	5	6	7	8	9	10	11
1.1	1.3	1.4	0.7	0.6	0.6	0.8	1.0	0.6	0.3	0.0

Back calculations of W' for the years 1934 through 1943 made by Ol' gave a standard deviation of 7.8 for the yearly Wolf numbers. It is clear that MacNish's method is essentially just a simplified analytical expression of the regression method.

Waldmeier (1946) derived forecasting formulas for the smoothed monthly relative spot numbers. If W_M is the maximum smoothed monthly Wolf number and W_{M+k} is the smoothed Wolf number k years from the maximum, then Waldmeier's formulas have the form

$$\left.\begin{aligned}
W_{M-1} &= -0.225W_M + 51.0,\\
W_{M-2} &= -0.072W_M + 26.0,\\
W_{M+1} &= 0.823W_M - 1.4,\\
W_{M+2} &= 0.686W_M - 4.8,\\
W_{M+3} &= 0.553W_M - 10.9,\\
W_{M+4} &= 0.380W_M - 5.2,\\
W_{M+5} &= 0.301W_M - 7.4.
\end{aligned}\right\} \quad (2.15)$$

In § 2 of this chapter Waldmeier's method was discussed, so that there is no reason to repeat his basic assumptions here. Waldmeier's formulas (2.15) are externally very similar to those of MacNish's method and especially to those of the regression method. However, the essential difference of Waldmeier's method is that in it all the Wolf numbers are expressed in terms of W_M, with the result that the errors are mainly determined by the errors in predicting W_M.

Waldmeier proceeded from his forecast maximum of $W_M = 139$ and his forecast epoch of maximum of 1947.6 to predict the following smoothed monthly Wolf numbers for the 18th solar cycle (the observed values are given in parentheses):

$$\begin{aligned}
&W_{M-1}\,(1946.6) - 89\ (93), && W_{M+3}\,(1950.6) - 66\,(84),\\
&W_M\ \ (1947.6) - 139\,(152), && W_{M+4}\,(1951.6) - 48\,(69),\\
&W_{M+1}\,(1948.6) - 113\,(136), && W_{M+5}\,(1952.6) - 30\,(31),\\
&W_{M+2}\,(1949.6) - 91\,(135), && W_{M+6}\,(1953.6) - 17\,(13).
\end{aligned}$$

These data indicate that Waldmeier's formulas do not make it possible to take into account fluctuations for the descending part of the 11-year sunspot cycle.

§ 9. A Procedure for Forecasting Yearly Wolf Numbers Within a Given 11-Year Cycle

We have now discussed practically all the methods for forecasting the yearly (or the equivalent smoothed monthly) Wolf numbers within a given 11-year cycle. These methods are based on Waldmeier's "eruption" hypothesis, with the exception, to some extent, of Gleissberg's method. Mayot's method (1947, 1951) has not been discussed, mainly due to the very low accuracy of this method and due to the fact that it is essentially similar to the other methods, so that it is of no real use in forecasting. Moreover, Mayot's method will be discussed in the next chapter in connection with medium-period forecasts, in which case it finds successful application.

In an evaluation of the practical application of forecasting methods, first all their advantages and shortcomings should be examined, then any methods which give very low accuracy or which are unsuitable for the given specific problem should be rejected, and finally it should be determined whether the errors of different methods compensate one another.

Since we are only concerned here with yearly relative spot numbers, the methods of Waldmeier and Gleissberg are unsuitable for our purposes. Moreover, MacNish's method in its original form is also unsuitable, and Ol's modification of this method has no real advantage over the regression method. Therefore, the following forecasting methods remain: 1) the cyclic-curve method, 2) the latitude method (Gnevyshev), 3) Kozik's method, and 4) the regression method.

The cyclic-curve method is complicated and limited in application, on the one hand, and gives a very low forecast accuracy, on the other. Since the latter disadvantage naturally does not compensate for the former, the application of this method for forecasting seems to us to be inadvisable.

An advantage common to the remaining three methods is their simplicity. On the other hand, the first two methods (the latitude method and Kozik's method) can be used to predict the yearly Wolf numbers for the descending part of the cycle only, whereas the regression method applies to both the descending and rising parts.

The latitude method pertains to a very limited portion of the descending part of the cycle ($\varphi < 14°$), and in this sense it represents only a kind of supplement to the other methods. This is particularly noticeable for high sunspot cycles, while for low sunspot cycles the latitude method applies over a much wider range, as was indicated in Chapter I, § 4.

Kozik's method gives Wolf numbers for practically the entire descending part of a given cycle. However, for low sunspot cycles the accuracies of forecasts made using this method are rather low, and it is most effective for high solar cycles.

A disadvantage of both the latitude method and Kozik's method is that these methods do not take into account fluctuations in solar activity. Thus, they both give Wolf numbers which are too low, particularly in the upper part of the descending portion of the cycle (the part closer to the maximum). The regression method gives the best accuracy for Wolf-number predictions a year in advance. This method, however, virtually does not apply to the lower part of the descending portion of the cycle (the part closer to the minimum). Contrary to the latitude method and Kozik's method, after a solar-activity fluctuation the regression method generally gives Wolf numbers which are too high. For forecasting several years in advance, the regression method is much less accurate.

Let us now consider directly a procedure for forecasting the yearly Wolf numbers within an 11-year solar cycle. To predict the Wolf numbers for the rising part of the cycle, the regression method is generally used. It should be noted that here a knowledge of at least the approximate time of the epoch of maximum for the given cycle (for example, gained with the aid of Waldmeier's method) will be helpful.

To predict the relative spot numbers for the descending part of the cycle, it is best to use a combination of the latitude method, Kozik's method, and the regression method, and to take the average of the values of W obtained using the different methods. In practice, for high cycles a combination of Kozik's method and the regression method is generally used, while for low cycles a combination of the latitude and regression methods is used. In the case of forecasts for the descending part of the sunspot cycle, the epoch of minimum must be known, and for this ultralong-range forecasts are generally made use of. Consequently, we will return to this problem in Chapter IV.

The preceding methods make it possible to forecast the Wolf numbers for almost the entire 11-year sunspot cycle. However, in practice, it is equally important to make high-accuracy forecasts one year in advance, and, as observed previously, the regression method is the most effective in this respect. On the other hand, since this method "overcompensates" for fluctuations in solar activity, we should not neglect the possibility of using Kozik's method, and in some cases the latitude method, to make the prediction more accurate.

Finally, still another comment on the methods to be used will be apropos. Since quite considerable unexpected jumps in the value of coefficient k of formula (1) in the Introduction are sometimes possible, even when the Wolf numbers are determined at the same station by the same observer, it is very desirable to make an annual comparison between the Zurich system of relative spot numbers and the system which is used directly to predict the future course of the current cycle.

Chapter III

MEDIUM-PERIOD FORECASTS OF SOLAR ACTIVITY

§ 1. General Considerations

As mentioned in the Introduction, medium-period forecasts of solar activity here refer to forecasts of the monthly and quarterly solar indexes. The following discussion will be limited to the prediction of relative spot numbers.

From Tables I and III of the Appendix we see that the quarterly, and especially the monthly, Wolf numbers fluctuate a great deal. Because of this, the prediction of these numbers is quite difficult, and when fluctuating solar activity is involved the forecast values usually have considerable error.

In contrast to the yearly Wolf numbers, which can be forecast several years in advance, the monthly relative spot numbers (provided smoothed values are not taken into account) can be obtained, using the methods available so far, no more than one month in advance. The situation is somewhat better for the quarterly Wolf numbers. However, the errors involved in all the methods of medium-period forecasting are rather high, so that these predictions are less useful than those of the yearly relative spot numbers.

Since several methods for forecasting the quarterly and monthly Wolf numbers are based on Mayot's fundamental concept, let us start by discussing his method, although actually this method was originally developed for the monthly relative spot numbers and should thus be considered in a somewhat later context. It should be noted that Mayot later extended his method to yearly Wolf numbers as well, but the accuracy of the method was in this case so low that this application hardly deserves consideration.

§ 2. Mayot's Method

Mayot (1947) starts from the assumption that a multiannual series of Wolf numbers is representable in the form

$$W(t)=F(t)+E, \qquad (3.1)$$

where $F(t)$ is some sum of trigonometric or exponential functions and E is some random quantity.

In this case the quantities W_i can be written as

$$\left.\begin{aligned} W_i &= a_1 W_{i-1} + a_2 W_{i-2} + \ldots + a_i W_0 + \varepsilon_i, \\ W_{i+1} &= a_1 W_i + a_2 W_{i-1} + \ldots + a_i W_1 + \varepsilon_{i+1}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ W_n &= a_1 W_{n-1} + a_2 W_{n+2} + \ldots + a_i W_{n-i} + \varepsilon_n. \end{aligned}\right\} \tag{3.2}$$

The coefficients $a_1, a_2, \ldots, a_i$ are determined by the least-squares method, assuming that the sum $\sum_{k=i}^{n} \varepsilon_k^2$ is minimized.

Originally Mayot's method was developed for the smoothed monthly Wolf numbers, which differ from the observed numbers in that they are much less subject to fluctuation. In calculations for past cycles this method gave the very low standard deviation of ±1.9. For forecasting, however, Mayot's method is much less accurate.

Since Mayot's formula for the prediction of the smoothed monthly Wolf numbers was incorrect (Vitinskii, 1956a), it will not be given here. However, the correct formula will be given below, during the discussion of the corresponding method for forecasting the smoothed monthly relative spot numbers.

Another factor deserves mention at this point. Mayot tried to predict the Wolf numbers for the same months in different years, and his back calculations of the relative spot numbers for January from 1896 to 1944 gave a standard deviation of ±9.2. However, when it came to forecasting, the error was once again considerably higher. Moreover, in this case the basic premise of Mayot was false, in that a yearly variation in Wolf number is by no means observed in every cycle, and also in that the variation is so slight that it can hardly be used as a basis for computations.

Mayot's method, as mentioned above, involves considerable error when used to forecast the quarterly and monthly Wolf numbers. However, since these numbers may deviate in either direction by 20 to 25%, due to random variations in coefficient k (as will be shown below), the accuracy of the forecast may still be adequate. Thus the application of Mayot's method for medium-period forecasting is completely justified. Finally, with respect to the forecasting of monthly relative spot numbers, Mayot's method is unique.

§ 3. Forecasts of Quarterly Wolf Numbers for the Next Quarter

Vitinskii (1956b, 1960a, 1961c) has proposed three methods for forecasting the quarterly Wolf numbers for the quarter to come, namely the regression method, Mayot's method, and the modified Mayot method. Let us first consider Mayot's method, the application of which, as noted in the previous section, involves the solution of a system of equations such as (3.2). Such a system of equations can be solved only if the coefficients of the normal equations are sufficiently separable. Consequently, it is advisable to utilize material covering the period up to the epoch of maximum or near it, since it is just at this epoch that the Wolf numbers are large enough to satisfy this requirement. Vitinskii used the Zurich quarterly Wolf numbers for the years 1944 through 1959. By solving a system of equations of the form of (3.2), he obtained the following formula, which can be used to forecast the quarterly Wolf numbers for the following quarter:

$$W_5=0.92W_4+0.04W_3+0.25W_2-0.24W_1. \qquad (3.3)$$

Back calculations for the years 1945—1959, made using relation (3.3), gave a standard deviation of ±24, with $\bar{W}_f$= 106 (a relative standard deviation of ±24%). It should be noted that the largest deviations between the computed and observed values correspond to the period of strong Wolf-number fluctuations. In such cases the relative error reached 47%.

The modified Mayot method is also based on Mayot's original assumption, but in contrast to Mayot's original method it does involve the use of the quarterly Wolf numbers themselves but rather the deviations of these from some mean curve. This was done in an attempt to improve the separation of the coefficients of the normal equations when using the Zurich data for the years 1940 through 1955.

The mean curve for the Wolf numbers was obtained as follows. Let us assume that the average length of a sunspot cycle is 11 years, as shown by

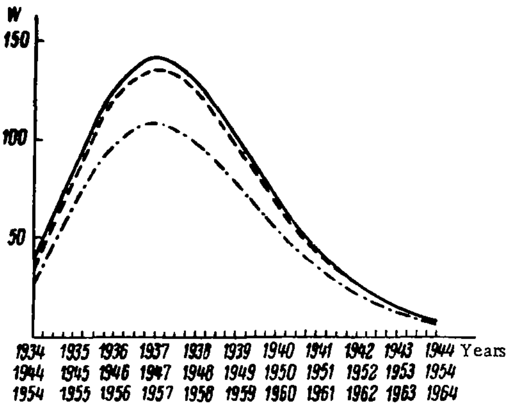

FIGURE 10

observations. Then, for such a cycle we can plot an average curve for the yearly relative spot numbers, using Stewart and Panofsky's formula (1.10) with a= + 7.1832 and b= + 1.2013 (Gleissberg, 1951a). Here, it is convenient to take the cycle intensity as W_M= 100, so that F= 0.3473. The quantity θ is reckoned from the epoch of minimum.

If we assume that a and b are constant for all cycles, then we can normalize this curve for each specific cycle, using the ratio of its intensity to W_M= 100. The cycle intensity is determined as the average of three yearly Wolf numbers, namely those measured at the epoch of maximum, one year before it, and one year after it. For the current cycle this curve was plotted using the forecasts of these numbers obtained using the methods discussed in Chapter II. Figure 10 shows examples of these curves for cycles 17, 18, and 19.

Next, let us read off the smoothed quarterly Wolf numbers $\bar{W}_i$ from the mean curves and then plot the differences between these numbers and the observed relative spot numbers W_i^0:

$$\Delta W_i = W_i^0 - \bar{W}_i. \tag{3.4}$$

If we solve a system of equations of the form

$$\Delta W_i = a_1 \Delta W_{i-1} + a_2 \Delta W_{i-2} + \ldots + a_i \Delta W_0 + \varepsilon_i \tag{3.5}$$

by the method of least squares, then we can obtain the required formula for the prediction of ΔW_i. Let us consider a solution of system (3.5) in six unknowns. In this case we obtain

$$\Delta W_7 = 0.64 \Delta W_6 + 0.17 \Delta W_5 + 0.20 \Delta W_4 - 0.22 \Delta W_1, \tag{3.6}$$

where ΔW_2 and ΔW_3 are negligible and so have been omitted. After reading off the corresponding value of $\bar{W}_i$ from the mean curve, we can obtain the quarterly Wolf number from the formula

$$W_7 = \bar{W}_7 + \Delta W_7. \tag{3.6a}$$

Back calculations of the Wolf numbers for the years 1941 through 1955, made using formulas (3.6) and (3.6a), gave a standard deviation of ±20, with an average quarterly Wolf number W_i of 61 (relative standard deviation of ±33 %).

The modified Mayot method, as the preceding figures show, has only a relatively low accuracy. This is to a large extent due to the fact that this method is more or less a combination of two methods. On one hand, it uses the Stewart-Panofsky curve, which is based on ultralong-range forecasts for the current cycle; on the other hand, it includes a forecast for the perturbed part ΔW_i. Consequently, the accuracy of this method should be expected to be lower than that of each of its component methods. Also, one of the defects of the modified Mayot method is that it is quite unsatisfactory for predicting the quarterly relative spot numbers during periods of strong fluctuation.

The regression method (Ol', 1954) for forecasting yearly Wolf numbers can also be applied successfully to the prediction of quarterly numbers, as a statistical analysis of the Zurich data for 19 incomplete solar cycles has shown. In order to apply the regression method, we require a quite accurate knowledge of the epochs of maximum and minimum of the solar cycles. The methods for forecasting these quantities will be discussed in Chapter IV, so that they will not be considered here. Let us just observe that the accuracy involved in determining the epochs of extremum up to one quarter is perfectly satisfactory.

An examination of the Zurich data for the rising parts of 19 solar cycles gave the following regression equations, correlation coefficients r, and standard deviations σ:

$$\left.\begin{array}{lll}
W_{m+1} = 1.08 W_m + 4 & r = +0.55 & \sigma = \pm 4 \\
W_{m+2} = 1.36 W_{m+1} + 3 & r = +0.70 & \sigma = \pm 6 \\
W_{m+3} = 0.69 W_{m+2} + 3 & r = +0.81 & \sigma = \pm 4 \\
W_{m+4} = 1.08 W_{m+3} + 5 & r = +0.71 & \sigma = \pm 8 \\
W_{m+5} = 1.42 W_{m+4} & r = +0.89 & \sigma = \pm 9
\end{array}\right\} \tag{3.7}$$

$$
\left.\begin{array}{lll}
W_{m+6}=1.20W_{m+5}+6 & r=+0.92 & \sigma=\pm 10 \\
W_{m+7}=1.11W_{m+6}+7 & r=+0.90 & \sigma=\pm 12 \\
W_{m+8}=1.13W_{m+7}+4 & r=+0.94 & \sigma=\pm 12 \\
W_{m+9}=0.98W_{m+8}+12 & r=+0.91 & \sigma=\pm 15 \\
W_{m+10}=1.17W_{m+9}-7 & r=+0.94 & \sigma=\pm 17 \\
W_{m+11}=0.96W_{m+10}+11 & r=+0.95 & \sigma=\pm 17 \\
W_{m+12}=1.26W_{m+11}-2 & r=+0.97 & \sigma=\pm 13 \\
W_{m+13}=1.09W_{m+12}-3 & r=+0.97 & \sigma=\pm 10 \\
W_{m+14}=1.18W_{m+13}-8 & r=+0.96 & \sigma=\pm 14 \\
W_{m+15}=0.92W_{m+14}+9 & r=+0.90 & \sigma=\pm 11 \\
W_{m+16}=1.40W_{m+15}-10 & r=+0.88 & \sigma=\pm 15 \\
W_{m+17}=1.20W_{m+16}+1 & r=+0.97 & \sigma=\pm 9
\end{array}\right\} \quad (3.7)
$$

It is clear from relations (3.7) that the forecasts of the quarterly Wolf numbers for the first four quarters are not reliable enough. For the entire rising part of the cycle, we have $\sigma=\pm 12$, with $\bar{W}_{i}=49$ (relative standard deviation of $\pm 24\%$).

Analogously, for the descending part of the cycle we obtain the following regression equations:

$$
\left.\begin{array}{lll}
W_{M+1}=0.92W_{M}-13 & r=+0.96 & \sigma=\pm 12 \\
W_{M+2}=0.86W_{M+1}+4 & r=+0.95 & \sigma=\pm 12 \\
W_{M+3}=0.94W_{M+2}+8 & r=+0.90 & \sigma=\pm 18 \\
W_{M+4}=0.92W_{M+3}+12 & r=+0.85 & \sigma=\pm 22 \\
W_{M+5}=0.72W_{M+4}+21 & r=+0.82 & \sigma=\pm 21 \\
W_{M+6}=0.76W_{M+5}+12 & r=+0.78 & \sigma=\pm 23 \\
W_{M+7}=0.90W_{M+6}+4 & r=+0.87 & \sigma=\pm 19 \\
W_{M+8}=0.80W_{M+7}+12 & r=+0.91 & \sigma=\pm 15 \\
W_{M+9}=0.90W_{M+8}+1 & r=+0.92 & \sigma=\pm 13 \\
W_{M+10}=0.79W_{M+9}+6 & r=+0.92 & \sigma=\pm 11 \\
W_{M+11}=0.84W_{M+10}+10 & r=+0.77 & \sigma=\pm 19 \\
W_{M+12}=0.82W_{M+11}+2 & r=+0.88 & \sigma=\pm 14 \\
W_{M+13}=0.76W_{M+12}+6 & r=+0.86 & \sigma=\pm 12 \\
W_{M+14}=0.67W_{M+13}+8 & r=+0.89 & \sigma=\pm 8 \\
W_{M+15}=0.98W_{M+14}-1 & r=+0.90 & \sigma=\pm 9 \\
W_{M+16}=W_{M+15} & r=+0.84 & \sigma=\pm 14 \\
W_{M+17}=0.79W_{M+16}+4 & r=+0.86 & \sigma=\pm 11 \\
W_{M+18}=0.74W_{M+17}+8 & r=+0.81 & \sigma=\pm 11 \\
W_{M+19}=0.65W_{M+18}+1 & r=+0.88 & \sigma=\pm 8 \\
W_{M+20}=1.10W_{M+19}-2 & r=+0.96 & \sigma=\pm 5 \\
W_{M+21}=0.88W_{M+20}-1 & r=+0.92 & \sigma=\pm 6
\end{array}\right\} \quad (3.8)
$$

$$\left.\begin{array}{lll} W_{M+22}=0.84W_{M+21}+1 & r=+0.92 & \sigma=\pm 6 \\ W_{M+23}=0.70W_{M+22}+1 & r=+0.83 & \sigma=\pm 5 \\ W_{M+24}=0.89W_{M+23} & r=+0.85 & \sigma=\pm 7 \\ W_{M+25}=0.70W_{M+24} & r=+0.88 & \sigma=\pm 6 \\ W_{M+26}=1.02W_{M+25} & r=+0.85 & \sigma=\pm 7 \end{array}\right\} \quad (3.8)$$

For the entire descending part, $\sigma = 14$, with $\bar{W}_i = 52$ (relative standard deviation of $\pm 27\%$).

A comparison of the accuracy of these three methods, on the basis of an analytic treatment of previous observations, shows that the regression method and Mayot's method involve smaller errors than the modified Mayot method. Moreover, if we take into account that for Mayot's method the error increases appreciably upon transition from back calculations to forecasts, then it is clear that the regression method should be considered as the most effective. Unfortunately, however, for solar cycles with long rising parts (longer than 17 quarters) or with descending parts longer than 26 quarters, only the ordinary and modified Mayot methods can be used for the quarters involved.

Let us also mention one technical detail which pertains to all the methods for predicting quarterly Wolf numbers for the quarter to come. Since the forecast must be made at the end of the preceding quarter, preliminary values of the quarterly relative spot numbers for 83 or 84 days (out of 90 or 91 days) are used. In general, this has little effect on the forecast accuracy, except in cases of strong solar-activity fluctuations.

Finally, data are now available which make possible an estimation of the accuracy of forecasts of the quarterly Wolf numbers made using the modified Mayot method. An analysis of the figures for the period from the first quarter of 1956 to the third quarter of 1960 gives a standard deviation of ± 27, with $\bar{W}_i = 162$, so that $\left(1-\frac{\sigma}{\bar{W}_i}\right)100\% = 84\%$. This is much higher than the value $\left(1-\frac{\sigma}{\bar{W}_i}\right)100\% = 67\%$ obtained from back calculations.

§ 4. Forecasts of Quarterly Wolf Numbers Two Quarters in Advance

For various practical purposes, especially for certain problems in geophysics and radiophysics, it is very important to forecast quarterly Wolf numbers a longer time in advance. A direct attack on this problem, by means of repeated or even double forecasting, leads to considerable error and is thus not practicable.

First, let us restrict our discussion just to forecasts of the quarterly relative spot numbers two quarters in advance. In addition, let us introduce the following quantities: the ordinary semiannual Wolf numbers W'_i; the semiannual Wolf numbers W''_i obtained when the half-years are shifted back one quarter (which will be called the special semiannual Wolf numbers); the observed quarterly Wolf numbers W^0_i; and the predicted quarterly Wolf numbers W_i.

Vitinskii (1960, 1961c) has proposed two ways of solving this problem. The first alternative is a prediction of the ordinary and special semiannual

Wolf numbers, which are then combined with the quarterly observed Wolf number in order to forecast the quarterly relative spot numbers two quarters in advance. In order to do this, the following formulas are used:

$$\left.\begin{aligned} W_{\mathrm{I}} &= 2W''_{\mathrm{I}} - 2W'_{\mathrm{II}} + W^{0}_{\mathrm{III}}, \\ W_{\mathrm{II}} &= 2W'_{\mathrm{I}} - 2W''_{\mathrm{I}} + W^{0}_{\mathrm{IV}}, \\ W_{\mathrm{III}} &= 2W''_{\mathrm{II}} - 2W'_{\mathrm{I}} + W^{0}_{\mathrm{I}}, \\ W_{\mathrm{IV}} &= 2W'_{\mathrm{II}} - 2W''_{\mathrm{II}} + W^{0}_{\mathrm{II}}, \end{aligned}\right\} \qquad (3.9)$$

where subscripts I and II of W'_i and W''_i indicate the corresponding half-years, while subscripts I through IV of W_i and W^0_i indicate the quarters of the year.

The second alternative consists in a prediction of the ordinary and special semiannual Wolf numbers, and also of the quarterly Wolf numbers, after which these are combined in order to forecast the quarterly relative spot numbers, using the formulas

$$\left.\begin{aligned} W_{\mathrm{I}} &= 2W''_{\mathrm{I}} - W_{\mathrm{IV}}, \\ W_{\mathrm{II}} &= 2W'_{\mathrm{I}} - W_{\mathrm{I}}\ , \\ W_{\mathrm{III}} &= 2W''_{\mathrm{II}} - W_{\mathrm{II}}, \\ W_{\mathrm{IV}} &= 2W'_{\mathrm{II}} - W_{\mathrm{III}}. \end{aligned}\right\} \qquad (3.10)$$

As mentioned previously, for this type of forecast an advance determination of the semiannual Wolf numbers is necessary. Either of two methods can be used for this, namely the regression method and Mayot's method.

On the basis of the Zurich data for 19 incomplete cycles, the following regression equations were obtained for the rising and descending parts of the solar cycle (the corresponding values of r and σ for the ordinary semiannual Wolf numbers are also given):

$$\left.\begin{aligned} W'_{m+1} &= 1.53W'_{m} + 3 & r&=+0.64 & \sigma&=\pm 5 \\ W'_{m+2} &= 1.30W'_{m+1} + 6 & r&=+0.59 & \sigma&=\pm 13 \\ W'_{m+3} &= 1.21W'_{m+2} + 12 & r&=+0.81 & \sigma&=\pm 14 \\ W'_{m+4} &= 1.28W'_{m+3} + 9 & r&=+0.89 & \sigma&=\pm 16 \\ W'_{m+5} &= 1.25W'_{m+4} + 7 & r&=+0.95 & \sigma&=\pm 14 \\ W'_{m+6} &= 1.01W'_{m+5} + 14 & r&=+0.92 & \sigma&=\pm 17 \\ W'_{m+7} &= 1.14W'_{m+6} + 2 & r&=+0.96 & \sigma&=\pm 14 \\ W'_{m+8} &= 1.37W'_{m+7} - 8 & r&=+0.85 & \sigma&=\pm 18 \end{aligned}\right\} \qquad (3.11)$$

$$\left.\begin{aligned} W'_{M+1} &= 0.84W'_{M} - 7 & r&=+0.93 & \sigma&=\pm 12 \\ W'_{M+2} &= 0.96W'_{M+1} + 1 & r&=+0.94 & \sigma&=\pm 15 \\ W'_{M+3} &= 0.79W'_{M+2} + 11 & r&=+0.84 & \sigma&=\pm 12 \\ W'_{M+4} &= 0.84W'_{M+3} + 2 & r&=+0.96 & \sigma&=\pm 10 \\ W'_{M+5} &= 0.89W'_{M+4} - 6 & r&=+0.95 & \sigma&=\pm 9 \\ W'_{M+6} &= 0.76W'_{M+5} + 5 & r&=+0.92 & \sigma&=\pm 9 \end{aligned}\right\} \qquad (3.12)$$

$$\left.\begin{aligned}
W'_{M+7} &= 0.87W'_{M+6} & r &= +0.89 & \sigma &= \pm 10\\
W'_{M+8} &= 0.58W'_{M+7} + 8 & r &= +0.80 & \sigma &= \pm 6\\
W'_{M+9} &= 0.85W'_{M+8} + 1 & r &= +0.73 & \sigma &= \pm 12\\
W'_{M+10} &= 0.77W'_{M+9} & r &= +0.89 & \sigma &= \pm 8\\
W'_{M+11} &= 0.82W'_{M+10} & r &= +0.91 & \sigma &= \pm 6
\end{aligned}\right\} \qquad (3.12)$$

An analysis of past data for the rising part gave a standard deviation of ± 14, with $\bar{W}'_i = 51$ (relative standard deviation of $\pm 27\%$). For the descending part of the cycle, the standard deviation was ± 10, with $\bar{W}'_i = 56$ (relative standard deviation of $\pm 18\%$). It should be noted that, according to (3.11), the forecasts of W'_i for the first two half-years of the rising part are unreliable.

For the special semiannual Wolf numbers, use of the same data led to the following regression equations (and values of r and σ) for the rising and descending parts of the solar cycle:

$$\left.\begin{aligned}
W''_{m+1} &= 1.33W''_{m} + 5 & r &= +0.58 & \sigma &= \pm 6\\
W''_{m+2} &= 1.72W''_{m+1} - 1 & r &= +0.81 & \sigma &= \pm 9\\
W''_{m+3} &= 1.70W''_{m+2} + 4 & r &= +0.93 & \sigma &= \pm 10\\
W''_{m+4} &= 1.32W''_{m+3} + 10 & r &= +0.94 & \sigma &= \pm 16\\
W''_{m+5} &= 1.20W''_{m+4} + 5 & r &= +0.96 & \sigma &= \pm 12\\
W''_{m+6} &= 1.17W''_{m+5} - 3 & r &= +0.95 & \sigma &= \pm 17\\
W''_{m+7} &= 1.05W''_{m+6} + 14 & r &= +0.95 & \sigma &= \pm 15\\
W''_{m+8} &= 1.20W''_{m+7} + 9 & r &= +0.84 & \sigma &= \pm 17\\
W''_{m+9} &= 0.98W''_{m+8} - 1 & r &= +0.80 & \sigma &= \pm 17
\end{aligned}\right\} \qquad (3.13)$$

$$\left.\begin{aligned}
W''_{M+1} &= 0.82W''_{M} & r &= +0.96 & \sigma &= \pm 10\\
W''_{M+2} &= W''_{M+1} & r &= +0.93 & \sigma &= \pm 14\\
W''_{M+3} &= 0.77W''_{M+2} + 10 & r &= +0.92 & \sigma &= \pm 12\\
W''_{M+4} &= 1.12W''_{M+3} - 17 & r &= +0.92 & \sigma &= \pm 13\\
W''_{M+5} &= 0.67W''_{M+4} - 9 & r &= +0.88 & \sigma &= \pm 12\\
W''_{M+6} &= 0.96W''_{M+5} - 9 & r &= +0.83 & \sigma &= \pm 13\\
W''_{M+7} &= 0.58W''_{M+6} + 10 & r &= +0.94 & \sigma &= \pm 7\\
W''_{M+8} &= 0.98W''_{M+7} - 1 & r &= +0.84 & \sigma &= \pm 11\\
W''_{M+9} &= 0.62W''_{M+8} + 5 & r &= +0.80 & \sigma &= \pm 10\\
W''_{M+10} &= 0.77W''_{M+9} - 2 & r &= +0.91 & \sigma &= \pm 6\\
W''_{M+11} &= 0.75W''_{M+10} & r &= +0.94 & \sigma &= \pm 4\\
W''_{M+12} &= 0.88W''_{M+11} - 2 & r &= +0.94 & \sigma &= \pm 4
\end{aligned}\right\} \qquad (3.14)$$

In this case the standard deviation for the rising part was ± 13, with $\bar{W}''_i = 51$ (relative standard deviation of $\pm 25\%$). For the descending part of the cycle, the standard deviation was ± 11, with $\bar{W}''_i = 52$ (relative standard deviation of ±21%). Relations (3.13) show that the forecasts for the first special half-year of the rising part are unreliable.

Mayot's method was applied to predictions of the ordinary and special semiannual Wolf numbers in the Zurich system for the years 1935 through 1959. In both cases these data ensured a satisfactory separation of the coefficients of the normal equations. To forecast the ordinary semiannual relative spot numbers, we can use the relation

$$W'_5 = 1.22W'_4 + 0.09W'_3 - 0.28W'_2 - 0.10W'_1. \qquad (3.15)$$

Back calculations using this formula gave a standard deviation of ±19, with $\bar{W}'_i = 82$ (relative standard deviation of ±23%).

The special semiannual Wolf numbers can be forecast using Mayot's method by means of the relation

$$W''_5 = 0.91W''_4 + 0.60W''_3 - 0.37W''_2 - 0.22W''_1. \qquad (3.16)$$

Here the standard deviation was ±21, with $\bar{W}''_i = 80$ (relative standard deviation of 26%).

It follows from the preceding that eight different methods can be used to forecast the quarterly Wolf numbers two quarters in advance:

1) the regression method for the ordinary and special semiannual Wolf numbers, using formulas (3.9), (3.11), (3.12), (3,13), and (3.14);

2) Mayot's method for the ordinary and special semiannual Wolf numbers, using formulas (3.9), (3.15), and (3.16);

3) the regression method for the ordinary and special semiannual Wolf numbers plus the modified Mayot method for the quarterly Wolf numbers, using formulas (3.10), (3.11), (3.12), (3.13), (3.14), (3.6), and (3.6a);

4) the regression method for the ordinary and special semiannual Wolf numbers plus Mayot's method for the quarterly Wolf numbers, using formulas (3.10), (3.11), (3.12), (3.13), (3.14), and (3.3);

5) the regression method for the ordinary and special semiannual Wolf numbers plus the regression method for the quarterly Wolf numbers, using formulas (3.10), (3.11), (3.12), (3.13), (3.14), (3.7), and (3.8);

6) Mayot's method for the ordinary and special semiannual Wolf numbers plus the modified Mayot method for the quarterly Wolf numbers, using formulas (3.10), (3.15), (3.16), (3.6), and (3.6a);

7) Mayot's method for the ordinary and special semiannual Wolf numbers plus Mayot's method for the quarterly Wolf numbers, using formulas (3.10), (3.15), (3.16), and (3.3);

8) Mayot's method for the ordinary and special semiannual Wolf numbers plus the regression method for the quarterly Wolf numbers, using formulas (3.10), (3.15), (3.16), (3.7), and (3.8).

Each of these methods [or combinations of methods] has its advantages and its defects. Let us first note that methods 3, 4, and 5 involve much lower errors than the other methods. For example, back calculations for the years 1945 through 1959 gave the following standard deviations σ and predictabilities

$$\left(1 - \frac{\sigma}{\bar{W}_i}\right) 100\%.$$

	σ	$\left(1-\frac{\sigma}{\bar{W}_t}\right) 100\%$		σ	$\left(1-\frac{\sigma}{\bar{W}_t}\right) 100\%$
1)	± 36	66%	5)	± 25	76%
2)	± 44	55	6)	± 36	63
3)	± 31	70	7)	± 32	67
4)	± 26	75	8)	± 41	59

Methods 1, 3, 4, and 5 are based on the regression method, a method which can be used only to determine time intervals away from the epochs of sunspot maximum and minimum (4.5 years and 6 years respectively). Thus, for example, these methods could not be applied to the back calculations for the period from the fourth quarter of 1953 to the fourth quarter of 1954.

Another important factor concerning the regression method also deserves mention, namely that this method cannot be used unless the epochs of extrema of the solar cycle are known in advance. There are methods available, however (see Chapters IV and V), which make it possible to determine these epochs to within a half-year.

The regression method is much less sensitive to strong fluctuations than Mayot's method, as is evident from a comparison of the corresponding relations.

We have already discussed some defects of Mayot's method. However, let us now consider one defect of the modified Mayot method which is particularly influential when this method is combined with Mayot's method for the semiannual Wolf numbers. The quarterly relative spot numbers calculated using this method are generally too high for the very beginning of the rising part of the cycle, and this lowers appreciably the quarterly numbers forecast two quarters in advance by means of method 6. In this case the very rough approximative correction of +44 should be introduced for the first two years of the cycle. This method reduces the standard deviation from ±38 to ±26, but it still represents only a very rough approximation.

Since Mayot's method involves higher errors than the regression method, it is only natural to assign higher weights to methods 1, 3, 4, and 5, which are based on the regression method. In order to increase the accuracy of the forecasts of the quarterly Wolf numbers two quarters in advance, it is necessary to take the average of the values obtained using the various methods, weights of 2 being assigned to methods 1, 3, 4, and 5, and weights of 1 being assigned to all the other methods. Back calculations for the years 1945 through 1959 have shown that this procedure will reduce the error appreciably. The standard deviation of these averaged predicted quarterly Wolf numbers was ±25, with $\bar{W}_t = 98$, giving a predictability of 75%. This value is quite acceptable, provided it is taken into account that the methods for forecasting the quarterly Wolf numbers for the quarter to come have practically this same predictability. However, the main disadvantage of such calculations, namely the large forecasting errors during times of strong fluctuations, still has its effect in this case.

§ 5. Forecasts of Smoothed Monthly Wolf Numbers

As mentioned in § 2 of this chapter, forecasts of smoothed monthly Wolf numbers were first developed by Mayot (1947). On the basis of the same initial data as Mayot (for the period from 1931 to 1944), Vitinskii derived the following formula for the prediction of the smoothed monthly Wolf numbers $\overline{W}_t$ one month in advance:

$$\overline{W}_5 = 0.99\overline{W}_4 + 1.22\overline{W}_3 - 1.70\overline{W}_2 + 0.49\overline{W}_1, \qquad (3.17)$$

to replace the incorrect formula of Mayot. Back calculations for these years, using formula (3.17), gave a standard deviation of ± 1.9. Such a low error makes this method seem very attractive, but, on the other hand, certain properties of the calculation of smoothed relative spot numbers lead us directly to a very grave difficulty.

We know that the quantities $\overline{W}_t$ are usually determined using formula (2) of the Introduction. It is clear from this formula, however, that the smoothed monthly Wolf numbers cannot be obtained earlier than six months before the given time. If all these Wolf numbers are calculated successively using Mayot's method, then the cumulative error will be so great that any advantages of this method (with regard to accuracy) will be reduced to naught. To overcome this obstacle, Vitinskii (1956c) proposed using the correlation between the observed and smoothed monthly Wolf numbers. The corresponding correlation coefficient is $r = +0.93$ and the regression equation is

$$\overline{W}_i = 0.98W_t + 2. \qquad (3.18)$$

This procedure naturally lowers the accuracy of the predictions made according to Mayot's method, especially in cases of solar-activity fluctuations, when the smoothed relative spot numbers obtained from (3.18) may be exaggerated considerably. However, the errors in this case are nevertheless lower than those entering in when the alternative method, proposed by Mayot, for predicting the Wolf numbers of the same months in different years is used. A purely technical detail should also be noted, namely that to make forecasts using this method it is quite sufficient to have preliminary monthly relative spot numbers for 23 to 27 days (out of 30 or 31 days).

An examination of the data for the period from January 1956 to October 1959 has shown that Mayot's method for forecasting the smoothed monthly Wolf numbers for the following month gives a standard deviation of ± 27, with $\overline{W}_i = 168$ (an average predictability of 84 %).

We have already mentioned that the use of equation (3.18) may introduce false fluctuations into forecasts of the smoothed relative spot numbers. This is illustrated by Figure 11, in which the solid curve represents the variation of smoothed monthly Wolf numbers for 1956 and 1957 as computed from observed quantities, while the dashed curve gives the Wolf numbers predicted using Mayot's method.

In order to eliminate this defect and (which is no less important) to increase the forecast range for smoothed monthly Wolf numbers, Vitinskii proposed a regression-interpolation method which has the advantages of simplicity and quite good accuracy. This method will now be described.

In the preceding section we discussed the regression method for ordinary and special semiannual Wolf numbers. Semiannual relative spot numbers can be considered as characteristic smoothed quantities, since they represent an average of six monthly values of the given solar index. Therefore, these numbers can be used to predict the smoothed monthly Wolf numbers. Once we know the semiannual Wolf number for the first half of a given year (referred to April), and once we have forecasted the semiannual Wolf number for the second half (referred to October), using the regression method, it is possible to obtain by interpolation predicted smoothed Wolf numbers for July, August, September, and October of this year. Analogously, on the basis of the special predicted and observed semiannual relative spot numbers, we can estimate in advance the smoothed Wolf numbers for October, November, December, and January, or April, May, June, and July. By means of the ordinary semiannual numbers, we can obtain values of the given index for January, February, March, and April.

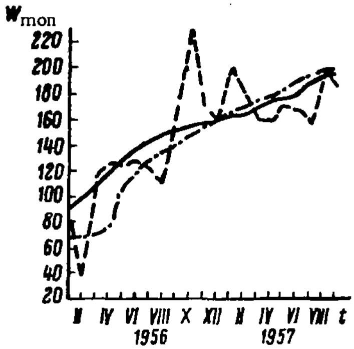

FIGURE 11

Thus, the regression-interpolation method makes it possible to predict the smoothed monthly Wolf numbers for the least favorable cases (February, May, August, and November) two months in advance, and sometimes even four months in advance (for March, June, September, and December). The variation of the predicted smoothed monthly relative spot numbers is much smoother than that obtained when Mayot's method is used. This is clear from Figure 11, in which the variation of the numbers computed using the regression-interpolation method is shown by the dash-dot line.

Back calculations for the period from January 1956 to October 1959, made using this method, gave a standard deviation of ±12, with $\bar{W}_t = 168$ (average predictability of 93%). If we take into account that the regression-interpolation method also increases the forecast range for the smoothed monthly Wolf numbers considerably, then the advantages of this method over Mayot's method become quite obvious.

Let us now consider the method, developed by Herrink (1958, 1959), for forecasting the smoothed monthly Wolf numbers up to the end of the current solar cycle. This method is based on a suggestion made by Anderson (1954) that there may exist a 169-year cycle of solar activity, since the Wolf numbers for the periods 1749 to 1785 and 1918 to 1954 show a quite close correspondence. Herrink compared the smoothed monthly relative spot numbers at the beginning of the rising parts of cycles 4 and 19, taking July 1784 as the beginning of the 4th cycle and April 1954 as the beginning of the 19th cycle. He obtained the following regression equation:

$$\bar{W}_{19} = 1.488\bar{W}_4 - 12.5. \tag{3.19}$$

Here, the subscripts indicate which 11-year cycle is referred to.

A forecast made according to this formula gave a standard deviation of only ±1.6. In an attempt to increase the forecasting accuracy, Herrink used the data for the period from April 1954 to October 1958 and obtained the new regression equation

$$\bar{W}_{19} = 1.527\bar{W}_4 - 13.4. \tag{3.20}$$

This equation differs only slightly from equation (3.19). Equation (3.20) can be used to forecast the smoothed monthly Wolf numbers up to the end of the 19th cycle.

Table 10 lists the smoothed monthly Wolf numbers predicted for 1957 and 1958 according to formula (3.19) (columns pred.) and for the years 1959 through 1967 according to formula (3.20). The observed values for 1957 and 1958 are also given (columns obs.). Starting with November 1966, formula (3.20) gives negative values, and Herrink replaced these by zeros.

TABLE 10

Predicted smoothed monthly Wolf numbers for 19th cycle (Herrink)

Month	1957		1958		1959	1960	1961	1962	1963	1964	1965	1966	1967
	pred.	obs.	pred.	obs.									
I	172	170	196	198	164	137	93	81	62	46	29	15	0
II	175	172	189	199	163	136	91	80	61	49	28	14	.0
III	182	177	187	203	168	129	89	79	59	50	23	11	0
IV	185	183	180	198	165	122	87	78	58	47	19	11	0
V	184	187	179	191	167	117	86	77	57	46	19	9	0
VI	188	189	178	189	163	111	86	76	55	43	16	6	0
VII	191	191	176	187	160	108	87	75	51	40	19	5	0
VIII	189	190	177	182	157	106	85	73	52	41	19	2	0
IX	191	194	178	183	153	103	83	71	50	36	16	1	0
X	196	194	174	181	147	100	81	70	49	33	17	1	0
XI	197	197	169	181	143	97	80	68	48	31	16	0	0
XII	197	197	167	180	140	95	82	64	46	28	15	0	0

For all its attractiveness, Herrink's method has the following two serious defects.

1. It presupposes the existence of a 169-year cycle, but does not take into account the 80-year to 90-year cycle, the existence of which has been quite reliably established. Consequently, its flexibility is somewhat limited; strictly speaking, it applies only to the 19th cycle and not even to all of it.

2. Herrink's assumption that the descending parts of cycles 4 and 19 have the same length is definitely an arbitrary one. Many authors have placed the epoch of minimum of the 20th cycle no later than 1966, and according to Ol' (1960) it will be 1965.2. Therefore, the values forecast for 1964 and 1965 are apparently too high.

Nevertheless, if suitably modified, Herrink's method can be used to predict the smoothed monthly Wolf numbers and the quarterly Wolf numbers not only for the 19th solar cycle but also for any other cycle, particularly for a cycle of higher intensity.

§ 6. Forecasts of Observed Monthly Wolf Numbers

As mentioned in the Introduction, the observed monthly Wolf numbers fluctuate greatly. Therefore, the prediction of monthly relative spot numbers, even just one month in advance, is a quite complicated problem. Forecasts with errors up to 25% may thus be considered quite acceptable, since the monthly Wolf numbers are uncertain within this margin.

The only method for forecasting the monthly Wolf numbers for the month to come is Mayot's method. Using the Zurich data for 1951 through 1956, Vitinskii (1960a) obtained the following equation for the prediction of monthly relative spot numbers:

$$W_5 = 0.81W_4 - 0.14W_3 + 0.51W_2 - 0.19W_1. \tag{3.21}$$

Back calculations for 1944 through 1956 made using this formula gave a standard deviation of ±22 (relative standard deviation of 27%).

Later, in order to ensure a more reliable application of Mayot's method, the following equation was derived from the Zurich data for 1954 through 1958:

$$W_5 = 1.32W_4 - 0.61W_3 + 0.82W_2 - 0.52W, \tag{3.22}$$

and it represents a better characterization of the current cycle. For back calculations this equation gave a standard deviation of ±22 with $\bar{W}_4 = 123$ (relative standard deviation of 18%).

Mayot's method gives the highest errors during periods of strong fluctuation in solar activity. Consequently, certain artificial procedures have been introduced to increase the forecast accuracy for the monthly Wolf numbers. First let us note that so far it is still impossible to foresee the onset of a strong fluctuation with any reliability at all, even with an accuracy up to one quarter. The methods described in the following thus only are intended mainly to predict the duration of a fluctuation.

An analysis of the statistical data shows that, except in rare cases, a sharp increase in solar activity is followed by a drop in activity during the next month. Therefore, to a first approximation, we can neglect such rises in activity and we can utilize for forecasting purposes only the general upward or downward trend of the activity curve for the given cycle.

The next approximation will consist in taking into account the development of long-lived sunspot groups, since fluctuations in solar activity are often determined by these groups. In Chapter I, § 5 the main features of the development of long-lived spot groups were described, and these features can be used to obtain purely qualitative estimates of the rate of drop of solar activity toward the next solar rotation.

Finally, still another approximation is possible, in view of the fact that active longitudes have a quite considerable effect on fluctuations in solar activity. A study of the processes related to active longitudes has shown that these are rhythmic, with an average period of 4 or 5 solar rotations (Vitinskii and Rubashev, 1957). Since, at any given time, one of the existing active longitudes predominates, therefore we can in general predict with some reliability approximately when a fluctuation in solar activity is to be expected. However, an important reservation must be made here. The active longitudes in the aforementioned reference were studied using the spot-group area as an index, and the behavior of this index is different from that of the Wolf numbers. Therefore, the conclusions obtained cannot be reliably applied to forecasting of the Wolf numbers rather than the areas.

Finally, let us estimate the accuracy of forecasts of the monthly Wolf numbers made for the following month, using Mayot's method. An analysis of the data for the period from July 1957 to September 1960 gives a standard deviation of ±28, with $\bar{W}_4 = 168$ (so that $\left(1 - \frac{\sigma}{\bar{W}_4}\right)100\% = 83\%$). This is somewhat better than the figures obtained for back calculations, a situation contrary to what is generally expected of Mayot's method for predicting the smoothed Wolf numbers. It should also be mentioned that so far no method is known which makes it possible to forecast the observed Wolf numbers two months or more in advance.

§ 7. The Analog Method for Forecasting Quarterly Wolf Numbers

A somewhat modified form of Herrink's method, which was described in § 5 of this chapter, makes it possible to forecast the quarterly Wolf numbers for the entire descending part of the current cycle. In view of the previously cited defects of this method, however, let us here proceed from basic premises which are somewhat different. First let us note the following two facts:

1) one of the most important characteristics of the solar cycle (Xanthakis, 1959) is the length of the rising part of the cycle;

2) the lengths of the descending parts of analog cycles usually differ from those of the prototype cycle, so that these lengths can be determined only by means of the existing methods for forecasting the epoch of minimum of a solar cycle.

Since the method proposed by Vitinskii (1960e, 1961c) starts by selecting an analog cycle for the given cycle, therefore this method will be called in the following the analog method. The analog cycles will be chosen according to two criteria: first, equality (or at least approximate equality) of the lengths of the rising parts of the test cycle and the given cycle; and second, the closest possible correlation between the quarterly Wolf numbers for the rising parts of the two cycles. The rising part of the test cycle must be no more than one quarter longer than the rising part of the given cycle. Even if all the correlation coefficients are high, still only the cycle with the very highest r is selected. If more than one cycle has the same [highest] correlation coefficient, then all these are used.

The adoption of this procedure, and the rejection of Herrink's basic premises, are completely justified. Actually, if the existence of a 169-year solar cycle is assumed, then the analogs of the 17th and 18th 11-year cycles are the 2nd and the 3rd cycles. However, Vitinskii has shown that the analog of the 17th cycle is really the 10th cycle, while that of the 18th cycle is the 11th cycle. Moreover, cycles 4 and 13 can equally well be considered as analogs of the 19th cycle.

In order to forecast the quarterly Wolf numbers for the descending part of the cycle, right to the end of the cycle, a regression equation must be derived from a comparison of these numbers for the rising parts of the given cycle and the analog cycle (as was done by Herrink for the smoothed monthly relative spot numbers). Back calculations for the descending parts of cycles 17 and 18, made using the regression equations obtained for these cycles, gave predictabilities of 63% and 72%, respectively. The difference in predictability is due to the fact that less strong cycles fluctuate more, and to the fact that the periods of strong fluctuations in different cycles seldom coincide to within one quarter.

On the basis of the analog method, Vitinskii obtained the following regression equations for the 19th cycle:

$$W_{19} = 1.49W_4 + 2, \tag{3.23}$$

$$W_{19} = 2.41W_{13} + 2, \tag{3.24}$$

where the subscripts indicate the number of the cycle.

Back calculations for the period from the first quarter of 1958 to the third quarter of 1960, made using formula (3.23), gave a relative standard deviation of 17%, while calculations using formula (3.24) gave a relative standard deviation of 16%. Since both analog cycles have approximately the same relevant characteristics, we took the average of the two as the forecast parameter. This was also reasonable because the 4th cycle has an anomalously long descending part (in contrast to the 13th cycle), and this could thus have caused the numbers predicted for the last years of the current cycle to be too high. After the averaging, back calculations for the period from the first quarter of 1958 to the third quarter of 1960 gave a relative standard deviation of 15%.

It is clear from this example that the analog method makes it possible to predict the quarterly Wolf numbers only for the descending part of the cycle. Since the descending part differs essentially from the rising part, therefore the accuracy of the forecast numbers should be improved as the descending part of the cycle develops; this can be done using additional data and modified regression equations.

If data for the descending part of the 19th cycle (for the third quarter of 1960) are introduced, then we obtain the following regression equations:

$$W_{19} = 1.39W_4 + 3, \tag{3.25}$$

$$W_{19} = 2.37W_{13} - 9. \tag{3.26}$$

These equations do not differ much from equations (3.23) and (3.24), but they are more reliable, since they reflect the tendency of the drop in solar activity during the current cycle.

TABLE 11

Forecast of quarterly Wolf numbers for 1958—1965 (according to Vitinskii)

Quarter	1958		1959		1960		1961	1962	1963	1964	1965
	p	p—o	p	p—o	p	p—o					
I	204	+18	168	-14	157	+42	88	62	67	38	21
II	183	+2	170	-11	127	+1	86	87	52	38	
III	220	+22	171	-27	108	-22	64	60	37	35	
IV	190	+16	162	+42	104		81	66	39	40	

Table 11 gives the predicted quarterly Wolf numbers (the averages of the values obtained using formulas (3.25) and (3.26)) for the period from the fourth quarter of 1960 to the first quarter of 1965. The epoch of minimum for the 20th cycle is taken, according to Ol' (1960), as 1965.2. The table also gives the quarterly relative spot numbers computed in the same way using formulas (3.23) and (3.24), for the period from the first quarter of 1958 to the third quarter of 1960 (columns p in the table), and also the deviations of these from the observed Zurich quarterly numbers (columns (p—o)).

It should be noted that the numbers given in the table for the years 1964 and 1965 are apparently too high, since even the descending part of the 13th cycle is longer than the predicted descending part of the 19th cycle.

Chapter IV

ULTRALONG-RANGE FORECASTS OF SOLAR ACTIVITY

§ 1. General Remarks

As mentioned in the Introduction, the ultralong-range forecasting of solar activity has had a great number of works devoted to it, many of these involving very complicated mathematics, and yet it is likely that this subject has provided the highest number of failures and disappointments in solar research. Many outstanding mathematicians (such as Schuster, Jewell, and Slutskii) have examined the problems related to ultralong-range forecasts, but these problems are still far from being solved, even today.

The first basic studies of ultralong-range forecasts were made at the end of the 19th century. However, the most intensive development of these methods took place at the beginning of this century. From almost the very beginning, two contradictory hypotheses were advanced, namely the superposition hypothesis and the "eruption" hypothesis. According to the superposition hypothesis, which was suggested by Wolf in about 1889, the curve of sunspot growth represents the result of a superposition of many periodic processes. In principle, this may yield a curve of any desired complexity. According to the "eruption" hypothesis, first advanced by Halm (1901), each 11-year sunspot cycle is considered to be a more or less independent eruption, and must be considered by itself.

We have already noted that the "eruption" hypothesis played a significant role in the development of methods for forecasting solar activity within a given 11-year cycle. However, when applied to ultralong-range forecasts, this hypothesis has had a somewhat negative effect. The superposition hypothesis, with all its defects, has served as a stimulus for the development of methods of ultralong-range forecasts. Although at present this hypothesis is of purely historical interest, still let us open by discussing some methods which are based on it. The most reasonable methods, those which were developed later, will be stressed here.

In this chapter some methods based on the properties of the 80-year to 90-year cycle and the 22-year cycle will also be discussed. These methods give the highest predictability. Schove's method, which is based on very extensive actual data, is especially interesting.

Finally, we will also consider methods for the ultralong-range forecasting of spot-group areas. Although these methods are essentially semiqualitative, they are nevertheless significant, since they represent a first attempt toward forecasting still another solar-activity index.

Forecasting the main characteristics of the next (the 20th) sunspot cycle is also a very interesting problem. The basic studies of this subject have therefore been collected to form a special chapter, the content of which is closely related to that of the present chapter.

§ 2. The Superposition Method

The superposition method is based on the superposition hypothesis, which was stated in the preceding section. Since this hypothesis maintains that the shape of the sunspot curve is determined not by one but by several periods, therefore the main efforts of various researchers were directed toward discovering all the possible periods which would give the best fit for the actual curve shape.

Wolf (according to Kimura, 1913) found, in addition to the period of 11.33 years, periods of 10, 8, 33, and 81 years. Thiele (1859) discovered, beside the main period, three additional periods, 9.805, 5.950, and 3.76 years in duration. Finally, Schuster made an important contribution toward defining the main periods of solar activity. Using the method of periodogram analysis which he developed, Schuster (1906) isolated a total of six periods, including one 11.125 years in duration. It should be noted, however, that he did not obtain a satisfactory fit for the Wolf-number curve.

The first attempt toward an ultralong-range forecast of the Wolf numbers was that of Kimura (1913). He used the yearly Wolf numbers for the years 1750 to 1911 in order to obtain 29 sinusoidal terms, which give a general representation of the relative spot numbers in the form

$$W=\sum a_n \sin(\varphi_n t + A_n), \tag{4.1}$$

where $n=1, \ldots, 29$, a_n is the amplitude, and A_n is the phase in the epoch of 1835.5. Although most of these periods can hardly be considered to exist, still Kimura's series gave a quite good fit for the curve of the relative spot numbers. For the years 1750 to 1800 the mean error in the curve fit was ± 10 (with a maximum of 28), and for the years 1800 to 1916 it was ± 6 (with a maximum of 13.6). It should be noted that one of the periods found by Kimura was 82.2 years.

Kimura predicted the yearly Wolf numbers for the years 1913 to 1950. His forecast for the 15th cycle was rather unsatisfactory (maximum Wolf number of 60 and epoch of maximum 1914, as compared with the actual values of 104 and 1917), but his predictions for the 16th and 17th cycles were acceptable:

16th cycle . . . . W_M = 85 (78); epoch of maximum 1927.5 (1928)
17th cycle . . . . W_M = 125 (114); epoch of maximum 1937 (1937)

The accuracy of Kimura's forecast for the 17th cycle is actually no lower than that of Waldmeier's forecast (W_M= 124 and epoch of maximum 1937.7). The difference between the two, however, is that Kimura made his forecast in 1913, while Waldmeier made his much later, in 1935. Figure 12 shows Kimura's results. The solid curve indicates the observed Wolf numbers and the dashed curve gives the numbers computed by Kimura.

It is not neccessary to list here the many works dealing with superposition methods, since they only differ from one another with respect to the mathematical apparatus used (periodogram analysis, harmonic analysis, Fourier functions, etc.). It is sufficient just to mention that the main

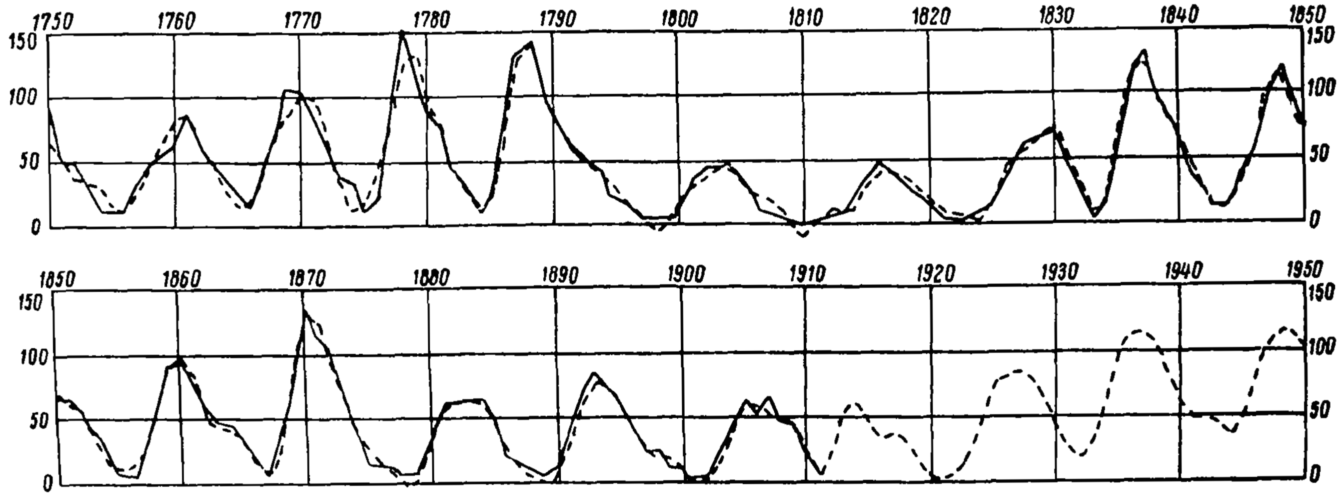
1750
1760
1770
1780
1790
1800
1810
1820
1830
1840
1850
150
100
50
0
1850
1860
1870
1880
1890
1900
1910
1920
1930
1940
1950
150
100
50
0

FIGURE 12

studies were those of Turner (1913a), Michelson (1913), Alter (1928), Oppenheim (1928), Stumpff (1930), Clayton (1939), and Anderson (1939). Most of these studies were definitely unsuccessful. A good illustration of this is Anderson's forecast, which presupposes the existence of a 312-year period. According to this forecast, the epoch of maximum for the 18th cycle falls in 1951 (actually it was 1947) and that of the 19th cycle falls in 1961 (actually it was 1957); the corresponding predicted maximum Wolf numbers are 75 (instead of 151) and 103 (instead of 190). The main reason for the failure of the superposition methods is their excessive formality. Too much attention was given to short periods (shorter than 11 years), while the significance of long periods for ultralong-range forecasts was definitely underrated. The most erroneous results were obtained when the Wolf-number curve was expanded into a Fourier series; on the other hand, periodogram analysis and the construction of a resultant curve using all the periods obtained may give results which are not bad, as is evident from Kimura's studies.

Consequently, the superposition method cannot give satisfactory forecasts of the Wolf numbers for subsequent cycles. Its real significance was just to draw attention to the study of long-period sunspot cycles and to stress the importance of ultralong-range forecasts of Wolf numbers.

§ 3. Gleissberg's Method

Just as Waldmeier was the first to give a successful forecast of the relative spot numbers for the current cycle, so Gleissberg occupies this same position among those who developed methods of ultralong-range forecasting.

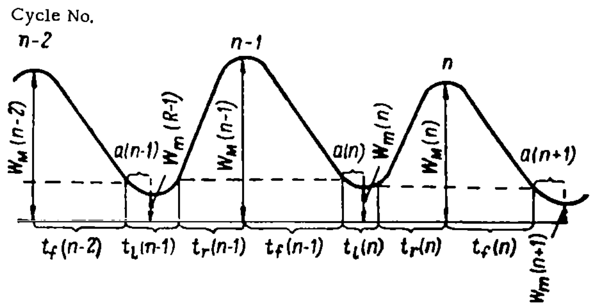

FIGURE 13

Gleissberg's method presupposes the existence of an 80-year to 90-year cycle of sunspots, the main features of which were discussed in Chapter I, § 8. Contrary to Waldmeier, however, Gleissberg maintained that successive cycles are not entirely independent, and that it is thus possible to forecast the next successive cycle and possibly later cycles as well.

Since it is very difficult to determine the epoch of beginning and the epoch of end of an 11-year cycle (Waldmeier, 1939), Gleissberg especially concentrated on the epochs when the Wolf number is equal to $\frac{1}{4}W_M$. Let us now introduce the following characteristics of the 11-year cycle (they are all shown in Figure 13):

W_M, the maximum Zurich smoothed monthly relative spot number;

t_r, the reduced length of the rising part of the cycle, defined as the time during which the smoothed monthly Wolf number increases from $\frac{1}{4}W_M$ to W_M (in months);

t_f, the reduced length of the descending part of the cycle, defined as the time during which the smoothed monthly Wolf number decreases from W_M to $\frac{1}{4}W_M$ (in months);

t_l, the period of low activity, defined as the time interval between the end of the reduced descending part of one cycle and the beginning of the reduced rising part of the next cycle (in months).

Next, on the basis of the Zurich data, let us obtain the values of these parameters (see Chapter II, Table 9). From the latter, the values of $W_M^{(4)}$, $t_r^{(4)}$, $t_f^{(4)}$ and $t_l^{(4)}$ may be computed, namely the averages of four successive values of W_M, t_r, t_f, and t_l, respectively. These numbers reflect quite clearly the variations in time of the main characteristics of the 11-year solar cycles. Although these variations are not particularly regular, and thus cannot be represented by exact mathematical formulas, they are nevertheless useful in ultralong-range forecasting. Consequently, ultralong-range prediction must be considered to be a probability problem.

If we use the data in Table 9 to calculate the quantities

$$\left.\begin{aligned} A &= t_r^{(4)} + 0.2W_M^{(4)}, \\ B &= t_r^{(4)} - 0.4t_l^{(4)}, \\ C &= t_r^{(4)} + 0.8t_f^{(4)}, \end{aligned}\right\} \qquad (4.2)$$

then we see that these quantities do not oscillate regularly and have a random distribution about their average values $\bar{A} = 55.4$, $\bar{B} = 16.4$, and $\bar{C} = 77.4$. Gleissberg assumed that these values vary only slightly when the following 11-year cycles are considered, and so for his subsequent computations he rounded these averages off to 55.5, 16.5, and 77.5. The distribution of differences between the actual values of A, B, and C and their average values is very close to Gaussian, with a mean error of $\delta = \pm 1.95$. Thus, $h = \frac{1}{\varepsilon\sqrt{2}} = 0.36$.

Therefore, the probability that the values of A, B, and C will differ from the average by no more than δ may be expressed as erf (0.36δ), where we define [the error function]

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy. \qquad (4.3)$$

Accordingly, Gleissberg (1952) derived the following probability laws:

I. the probability that $t_r^{(4)} + 0.2W_M^{(4)}$ for any two successive cycles lies between $55.5 - \delta$ and $55.5 + \delta$ may be expressed as **erf (0.36δ)**;

II. the probability that $t_r^{(4)} - 0.4t_l^{(4)}$ for these same cycles lies between $16.5 - \delta$ and $16.5 + \delta$ may be expressed as **erf (0.36δ)**;

III. the probability that $t_r^{(4)} + 0.8t_f^{(4)}$ for these cycles lies between $77.5 - \delta$ and $77.5 + \delta$ may be expressed as erf (0.36δ).

Since none of these probability laws establishes a correlation between different sets of cycles, they cannot be used to predict future cycles; to do this, an additional law is required. Also, since the quantity $t_r^{(4)}$ enters into all three laws, it was convenient to investigate the variation of this quantity from one cycle to the next. If this variation obeys the law of random error, then the probability that the variation amplitude is not greater than δ may be expressed as erf (0.16δ).

It will be useful to know what the probability is that $t_r^{(4)}$ will change by an amount δ. This probability can be obtained by subtracting the probability that $t_r^{(4)}$ will change by no more than $\delta-0.5$ from the probability that it will change by no more than $\delta+0.5$. Thus, the fourth probability law states that:

IV. the probability $P(\delta)$ that $t_r^{(4)}$ will change by an amount δ is

$$P(\delta) = \mathrm{erf}\,(0.16\delta + 0.08) - \mathrm{erf}\,(0.16\delta - 0.08). \qquad (4.4)$$

Table 12 lists some values of $P(\delta)$ and $Q(\delta)$. The meaning of the function $Q(\delta)$ will be explained below.

TABLE 12

Functions $P(\delta)$ and $Q(\delta)$ (according to Gleissberg)

δ	$P(\delta)$	$Q(\delta)$	δ	$P(\delta)$	$Q(\delta)$	δ	$P(\delta)$	$Q(\delta)$
0	0.09	0.50	5	0.10	0.99	10	0.01	1.00
1	0.18	0.69	6	0.07	1.00	11	0.01	1.00
2	0.16	0.85	7	0.05	1.00	12	0.01	1.00
3	0.14	0.94	8	0.04	1.00	$\geqslant 13$	0.00	1.00
4	0.12	0.98	9	0.02	1.00			

Gleissberg used these four probability laws to predict the 18th cycle. Since, according to Table 9, the next value of $W_M^{(4)}$ will be greater than 91.7 (it is approximately 110), therefore the value of W_M for the 18th cycle will be greater than 145. Since the sum of the last three quantities in the second column of Table 9 is 302.7, the sum of the four numbers W_M for cycles 15 through 18 will be greater than 447.7. Consequently, $W_M^{(4)}$ (for cycles 15 through 18) should be greater than 111.9, that is, for these cycles we will have $t_r^{(4)} + 0.2W_M^{(4)} > t_r^{(4)} + 22.5$. On the other hand, it follows from the eighth column of Table 9 that the next value of $t_r^{(4)}$ will not exceed 33. Thus, W_M of the next cycle will be greater than 145 if the following conditions are met:

$$\left.\begin{array}{l} t_r^{(4)} = 33 \text{ and } t_r^{(4)} + 0.2W_M^{(4)} \geqslant 55.5, \\ t_r^{(4)} = 32 \text{ and } t_r^{(4)} + 0.2W_M^{(4)} \geqslant 54.5, \\ t_r^{(4)} = 31 \text{ and } t_r^{(4)} + 0.2W_M^{(4)} \geqslant 53.5, \text{ etc.} \end{array}\right\} \qquad (*)$$

The probability that $t_r^{(4)}$ will change by an amount δ was denoted as $P(\delta)$. Since the last value of $t_r^{(4)}$ was 33 and since $t_r^{(4)}$ now decreases, $P(\delta)$ represents the probability that the next value of $t_r^{(4)}$ will be $33 - \delta$.

Let us now consider the probability that $t_r^{(4)} + 0.2W_M^{(4)}$ will be greater than $55.5 - \delta$. This probability is the sum of the probabilities that $t_r^{(4)} + 0.2W_M^{(4)}$ will lie between 55.5−δ and 55.5 or between 55.5 and ∞. The first probability, according to law I, may be represented as $^1/_2$ erf (0.36δ), while the second probability is $^1/_2$. Therefore, the probability that $t_r^{(4)}+0.2W_M^{(4)}$ will be greater than 55.5−δ is given by the function

$$Q(\delta)=\frac{1}{2}\operatorname{erf}(0.36\delta)+\frac{1}{2}, \qquad (4.4a)$$

some values of which are listed in Table 12.

Let us now calculate the probability that one of the conditions (*) will be met. This probability is given by the expression $\sum_{\delta=0}^{\infty} P(\delta)Q(\delta)$, and if we take the values of $P(\delta)$ and $Q(\delta)$ from Table 12 we find that the probability that W_M for the 18th cycle will exceed 145 is 86%. For such a high W_M, it can be expected that the reduced time t_r of the rising part of the 18th cycle will be very short. The average of all the t_r for cycles 1 through 17 is 35 months. The sum $t_r(15)+t_r(16)+t_r(17)=98$. If we assume that for the 18th cycle $t_r<32$, then $t_r(15)+t_r(16)+t_r(17)+t_r(18)<130$ and consequently $t_r^{(4)}\leqslant 32$.

As shown previously, the probability that the next $t_r^{(4)}$ will be equal to 33−δ is $P(\delta)$. Accordingly, the probability that $t_r^{(4)}$ will not be greater than 32 is $\sum_{\delta=1}^{\infty} P(\delta)$, and this sum is obviously equal to $1-P(0)$. Thus, from Table 12 we find that the probability that in the 18th cycle $t_r<32$ months is 91%.

The period of low activity t_l of the preceding cycle is less than 40 months. Also, the sum $t_l(15)+t_l(16)+t_l(17)=136$, so that taking into account our assumption that $t_l(15)+t_l(16)+t_l(17)+t_l(18)\leqslant 176$ (that is, that the next value of $t_l^{(4)}\leqslant 44$) we will obtain

$$t_r^{(4)}-0.4t_l^{(4)}\geqslant t_r^{(4)}-17.5.$$

By reasoning which is analogous to that for W_M, we find the probability that the next value of t_l will be less than 40. The latter is given by the sum $\sum_{\delta=0}^{\infty} P(\delta)Q(\delta+1)$. Table 12 may now be used to obtain a probability of 93% that the period of quiet activity will be less than 40 months.

Let us now compare the predicted and observed values of the main characteristics of the 18th cycle:

	Predicted	Observed
W_M	145	152
Epoch of maximum	1948.3	1947.5
t_r	32	21
t_l	40	37

The predicted maximum Wolf number shows a very good agreement with the observed value. However, the values obtained for the basic time characteristics of the cycle (especially for t_r) cannot, as will be shown in the following, be considered very successful.

On the other hand, it is for just these most unreliable predictions that Gleissberg claimed the highest probability of forecasting reliability. Thus, it may be concluded that the probability of reliability of the various sunspot-cycle parameters claimed by Gleissberg is, to a large extent, unjustified.

Sometimes the evaluation may even lead to a deceptive picture of the reliability of the forecast. The main feature of Gleissberg's method is its use of the properties of the 80-year to 90-year cycle. However, this method bases itself on the 80-year to 90-year cycle of the parameter $t^{(4)}_r$, and the variation of this parameter is not regular enough to permit a reliable extrapolation to the next cycle.

Subsequently, Gleissberg (1951b) modified his method somewhat, although the modification, as will be shown below, was not a fundamental one. Let us briefly outline this modification here, without entering into the probability theory involved (since the latter theory is not pertinent to the discussion), using the 19th cycle as an example. In addition to the previously used cycle characteristics, Gleissberg also introduces the minimum Wolf number W_m and its average over four successive cycles $W^{(4)}_m$. This quantity may be correlated with $t^{(4)}_r$ using the expression

$$t^{(4)}_r + 1.42W^{(4)}_m = 41.85. \tag{4.5}$$

He next defines the parameter

$$a = 0.375t_l + 0.005t_l^2. \tag{4.6}$$

Then, by extrapolating the $t^{(4)}_r$ curve, Gleissberg takes the extrapolated value of $t^{(4)}_r$ as 30 and uses the third formula of (4.2) to find that for the 18th cycle $t^{(4)}_f(18) = 59$. Thus, since $t_f(15) + t_f(16) + t_f(17) = 149$, we have $t_f(18) = 87$. Since $W_M(18)$ occurred in May 1947, therefore $t_f(18)$ should end in August 1954. Then since we took $t^{(4)}_r(19) = 30$, we find from the second formula that $t^{(4)}_l(19) = 34$. Also, $t_l(16) + t_l(17) + t_l(18) = 121$, so that $t_l(19) = 15$ and finally from (4.6) we have $a(19) = 7$; thus the minimum should occur in March 1955.

If we take $t^{(4)}_r(19) = 30$, then from $t_r(16) + t_r(17) + t_r(18) = 87$ we have $t_r(19) = 33$. Then $t_f(18) + t_l(19) + t_r(19) = 135$, and thus the maximum of the 19th cycle should be expected in August 1958. It follows from formula (4.5) that $W^{(4)}_m(19) = 8.3$. Since we know that $W_m(16) + W_m(17) + W_m(18) = 16.7$, we find that $W_m(19) = 16.5$. Finally, from the first formula of (4.2), we find that $W^{(4)}_M(19) = 127.5$, and since $W_M(16) + W_M(17) + W_M(18) = 349.1$ we have $W_M(19) = 160$.

Gleissberg's method also gives $t_f(19)$. From the third formula of (4.2), we find that $t^{(4)}_f(19) = 59.3$. Since $t_f(16) + t_f(17) + t_f(18) = 161$, we have $t_f(19) = 76$, so that $\frac{1}{4}W_M$ should be expected in January 1965.

There are now enough data available to evaluate the accuracy of Gleissberg's forecast for the 19th cycle. It should be mentioned that a more refined forecast for this cycle, made later by Gleissberg (1953) using a different method, is much inferior to the one just described. For example, according to this later forecast, $W_M(19) = 130$. It should also be kept in mind that Gleissberg used smoothed monthly Wolf numbers and the epochs of extrema corresponding to these.

The following data provide a comparison between the predicted and observed characteristics for the 19th cycle:

	Predicted	Observed
Epoch of minimum	1955.2	1954.5
W_m	16.5	3.6
Epoch of maximum	1958.7	1958.1
W_M	160	202

The epochs of extrema were predicted with satisfactory accuracy, but the forecast cycle height was much too low.

Finally, let us consider the errors involved in forecasting for 11-year cycles. Since the variation of $t_r^{(4)}$ is not actually regular, and since it is just this parameter upon which the entire forecast is based, therefore very small errors in $t_r^{(4)}$ will lead to quite large errors in the other predicted parameters. An error of ± 1 in $t_r^{(4)}$ results in the following errors:

$$t_r - \pm 4, \ W_M - \pm 20, \ t_l - \pm 10, \ t_f - \pm 5, \ a - \pm 4, \ W_m - \pm 3.$$

One other important shortcoming of Gleissberg's method and of its modification is that it takes virtually no account of the properties of the 22-year cycle and of the supersecular variation of solar activity. This method thus lacks internal control and is practically quite one-sided. As indicated previously, the probability evaluations of this method are very deceptive, and thus cannot replace internal control. Nevertheless, Gleissberg's method can still be used successfully in combination with other methods, and it has not lost its practical significance.

§ 4. The Method of Ol'

In 1949 Ol' developed a method for ultralong-range forecasting which is based on the assumption that the main periodicities governing the development of solar activity are the 11-year cycle, the 22-year cycle, the 80-year to 90-year cycle, and the supersecular variation (Ol', 1949a, 1949b). Let us now consider this method as it applies to the 19th cycle.

The method of Ol' uses the following parameters of the 11-year cycle:

W_M, the maximum yearly Wolf number;
$\sum W$, the sum of the yearly Wolf numbers for the cycle;
$\sum_1 W$, the sum of the yearly Wolf numbers for the rising part;
$\sum_2 W$, the sum of the yearly Wolf numbers for the descending part;
t, the length of the rising part (in years);
τ, the length of the descending part (in years);
T, the duration of the cycle (in years).

The superscript (4) will indicate the smoothing of a parameter over four cycles.

The point of departure for this method is the $W_M^{(4)}$ curve shown in Figure 14. An extrapolation of this curve, taking into account the supersecular variation (the dashed lines in Figure 14), gives a value of $W_M^{(4)} = 127$ for 1943. Thus, since $W_M(16) + W_M(17) + W_M(18) = 344$, we have $W_M(19) = 164$.

Now let us turn to the other parameters of the 11-year cycle. The quantities $\sum W$ and W_M are related by the formula

$$\sum W = -103 + 8.60 W_M - 0.0236 W_M^2. \qquad (4.7)$$

Since we know that $W_M(18) = 152$, we can use formula (4.7) to find $\sum W(18) = 660$. Then, since $\sum_1 W(18) = 279$, we have $\sum_2 W(18) = 381$.

In order to obtain the total duration T_{18} of the cycle, let us introduce the formula for the linear regression between $W_M^{(4)}$ and $\frac{\sum W^{(4)}}{T}$:

$$\frac{\sum W^{(4)}}{T} = 0.455 W_M^{(4)} + 1.7. \qquad (4.8)$$

For $W_M^{(4)} = 112$, we obtain $\frac{\sum W^{(4)}}{T} = 52.6$, that is $\left(\frac{\sum W}{T}\right)_{18} = 69.4$ and $T_{18} = 9.5$. Thus, since $t_{18} = 3.1$, we have $\tau_{18} = T_{18} - t_{18} = 6.4$.

The value of τ_{18} can be found by other methods. The linear regression between $\sum W^{(4)}$ and $\frac{\sum_2 W^{(4)}}{\tau}$ may be expressed by the formula

$$\frac{\sum_2 W^{(4)}}{\tau} = 0.0737 \sum W^{(4)} + 3.0. \qquad (4.9)$$

For cycles 15 through 18, $\sum W^{(4)} = 531$, so that $\frac{\sum_2 W^{(4)}}{\tau} = 42.0$. Thus, we obtain $\left(\frac{\sum_2 W}{\tau}\right)_{18} = 52.1$ and $\tau_{18} = 7.3$.

As noted in Chapter I, Waldmeier gave a formula for the regression between $Q = \frac{t}{\tau}$ and W_M. For even cycles this is

$$Q = \frac{15.64 - 5.81 \log W_M}{3.0 + 0.03 W_M}. \qquad (1.7)$$

Thus, for $W_M = 152$, we obtain $Q = 0.39$, that is, $\tau_{18} = 7.9$.

Let us take the average of these values for τ, namely $\tau_{18} = 7.2$. The total duration of the 18th cycle is then

$$T = 3.1 + 7.2 = 10.3,$$

that is, this cycle should end in the middle of 1954. It turned out that this forecast of the epoch of minimum for the 19th cycle was excellent.

Let us now consider the direct prediction of the basic parameters of the 19th cycle. Previously we obtained $W_M(19) = 164$, and from regression equation (4.7) we obtain $\sum W(19) = 670$. The following equation for the regression between $\sum W$ for an even cycle and the following odd cycle (Gnevyshev and Ol', 1948) may now be used:

$$\sum W_{\text{odd}} = 0.844 \sum W_{\text{even}} + 152. \qquad (4.10)$$

For $\sum W(18) = 660$, we find that $\sum W(19) = 709$. The average of the different values obtained for $\sum W(19)$ is $\sum W(19) = 690$.

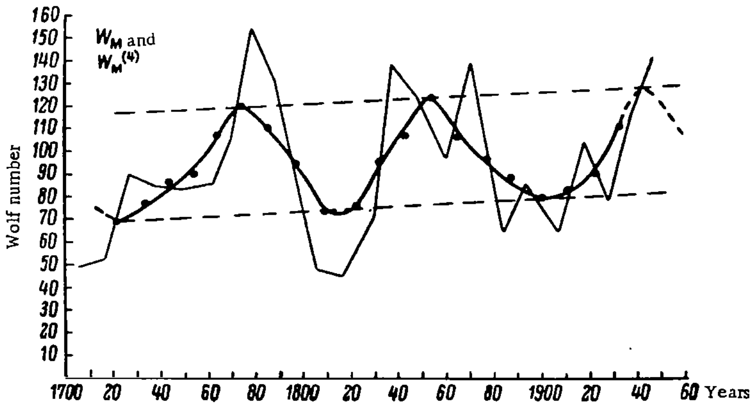

FIGURE 14

The quantity $\sum_2 W(19)$ is found using the parabolic regression formula

$$\sum_2 W = 13 + 0.21 \sum W + 0.000536 (\sum W)^2. \tag{4.11}$$

For $\sum W(19) = 690$, we obtain $\sum_2 W(19) = 413$. The following formula for the regression between $\sum_2 W^{(4)}$ and $\sum W^{(4)}$ can also be used:

$$\sum_2 W^{(4)} = 0.707 \sum W^{(4)} - 96. \tag{4.12}$$

Since for cycles 16 through 19 $\sum W^{(4)} = 592$, therefore we have $\sum_2 W^{(4)} = 322$ and $\sum_2 W(19) = 358$. The average value is $\sum_2 W(19) = 385$, so that we obtain $\sum_1 W(19) = 305$.

The duration of the 19th cycle can be determined using formula (4.8). For $W_M^{(4)} = 127$, we have $\frac{\sum W^{(4)}}{T} = 59.5$, that is $\left(\frac{\sum W}{T}\right)_{19} = 72.6$ and $T = 9.5$. In addition, formula (4.9) gives $\frac{\sum_2 W^{(4)}}{\tau} = 46.6$, that is, $\left(\frac{\sum_2 W}{-\tau}\right)_{19} = 56.0$ and $\tau_{19} = 6.9$. Consequently, $t_{19} = 2.6$ and $Q = 0.38$, which is close to the values obtained using Waldmeier's equation (1.8) for odd cycles.

Thus, Ol' predicted the following values for the basic parameters of the 19th solar cycle:

$$W_M = 164; \quad \sum W = 690; \quad \sum_1 W = 305; \quad \sum_2 W = 385; \quad t = 2.6;$$
$$\tau = 6.9; \quad T = 9.5.$$

Later, after taking into account more of the descending part of the cycle, Ol' (1954) obtained the following improved forecast for the 19th cycle:

$$W_M = 164; \quad \sum W = 730; \quad \sum_1 W = 278; \quad \sum_2 W = 472; \quad t = 2.4;$$
$$\tau = 7.2; \quad T = 9.6.$$

There are not enough data as yet for an evaluation of all these forecasts. However, it is interesting to compare the forecast and observed values of the quantities known so far:

$$W_M = 190; \quad \sum_1 W = 374; \quad t = 3.4.$$

The first forecast was somewhat more successful, the highest deviation being observed for the length of the rising part of the cycle. Interestingly enough, when the descending part was taken into account, the forecast was not improved using this method (as was also the case in the preceding section, using Gleissberg's method). This stresses once again the fact, pointed out in Chapter I, that for an 11-year cycle the length of the rising part of the cycle is of decisive significance.

Finally, let us discuss the statistical justification of the regularities made use of in the method of Ol'. If we assume that in this case we have only small samples, then the significance of the correlation coefficients can be evaluated by means of Romanovskii's criterion (1947):

$$|r| \sqrt{n-1} \geqslant 3. \tag{4.13}$$

Ol' obtained the following correlation coefficients:

$$\left.\begin{aligned} &r\left[\sum W_M^{(4)}, \frac{\sum_2 W^{(4)}}{\tau}\right] = 0.983, \\ &r\left[W_M^{(4)}, \frac{\sum W^{(4)}}{T}\right] = 0.983, \\ &r[W_M^{(4)}, \sum_2 W^{(4)}] = 0.983, \end{aligned}\right\} \quad n = 19$$

$$r[\sum W_{\text{odd}}, \quad \sum W_{\text{even}}] = 0.950 \quad n = 10.$$

It is evident that the first three coefficients definitely satisfy Romanovskii's condition and are thus significant. However, due to the smallness of n (n=10), the validity of the last coefficient cannot be inferred using Romanovskii's condition.

The method of Ol' is superior to Gleissberg's method, first, by virtue of its relative simplicity and, second, because of its internal control. The latter factor is particularly important, since it increases considerably the reliability of the forecast.

§ 5. Methods Related to the Properties of the 22-Year Cycles

Kopecký (1950b) proposed two methods, both related to the properties of the 22-year cycles, for the ultralong-range forecasting of Wolf numbers. The first of these methods is associated with a property of the 22-year cycles which was established by Gnevyshev and Ol' (1948), namely that there exists a close correlation between the characteristics of the odd and even 11-year cycles constituting a 22-year cycle (see Chapter I, § 7).

For the years with the best correlations between the Wolf numbers for the even and odd cycles, Kopecký (1950a) calculated the following coefficients for conversion from the relative spot numbers for even cycles to those for odd cycles: 2 — 1.10, 3 — 1.30, 4 — 1.30, 5 — 1.20, where the numbers preceding the dashes represent the number of the year in the 11-year cycle, measured from the epoch of minimum. Accordingly, the correlation coefficients are: for year 2, $+0.521 \pm 0.186$; for year 3, $+0.686 \pm 0.135$; for year 4, $+0.971 \pm 0.014$; and for year 5, $+0.788 \pm 0.097$. Kopecký (1950b) used these correlations to obtain the following yearly Wolf numbers for the 19th cycle: for year 2, 123.6; for year 3, 208.5; and for year 4, 165.3. If we take into account that the epoch of minimum of the 19th cycle occurred in 1954, then the 2nd year of the cycle was 1956. And so this forecast was quite successful. The height of the maximum of the 19th cycle was determined with an error of only 10%.

Let us now try to determine these numbers using the conversion coefficients, in which case we obtain: for year 2, 102; for year 3, 197; and for year 4, 167. Thus this method, which appears at first glance to be so primitive, gives a much improved accuracy, a value of W_M= 197 in comparison with the observed 190.

The second method proposed by Kopecký is based on the 80-year to 90-year variation in the principal characteristics of the 22-year cycles. Let us introduce the following parameters describing a 22-year solar cycle: ΣW_M, the sum of the maximum Wolf numbers of the Hale pair of 11-year cycles; and ΣW_a, the sum of the yearly relative spot numbers for the 22-year cycle. These sums are given in Table 13.

The table shows that two maxima of $\sum W_M$ and $\sum W_a$ occur in cycles 1 and 4, indicating a quite distinct 80-year to 90-year cycle in these parameters as well. Therefore a third maximum may be expected in the 9th 22-year cycle. If we assume that the values of $\sum W_M$ and $\sum W_a$ in the 9th cycle will be equal to their averages in the 1st and 4th Hale cycles, then we can estimate the height $W_M(19)$ of the 19th cycle. Observations give W_M (18)= 151.6 and

Table 13 gives $\sum W_M = 261.5$, so that we obtain $W_M(19) = 110$. Analogously, observations give $\sum W_a(18) = 702$ and Table 13 gives $\sum_{18}^{19} W_a = 1267$, so that $\sum W_a(19) = 565$.

There is a fairly close correlation between $\sum W_a$ and W_M, as determined by the regression equation

$$\sum W_a = 4.65 W_M + 62.5. \qquad (4.14)$$

This relation gives $W_M(19) = 108$. Obviously, Kopecký's forecast according to the second method is rather unsuccessful. This may be due to the fact that this method does not take into consideration the supersecular variation of solar activity, which affects $\sum W_M$ and $\sum W_a$ to some extent, as Table 13 shows. Moreover, the cyclic oscillations of $\sum W_M$ and $\sum W_a$ over many years are less regular than the variation in $W_M^{(4)}$.

TABLE 13

Parameters of 22-year solar cycles (according to Kopecky)

Number of Hale cycle	ΣW_M	ΣW_a	Number of Hale cycle	ΣW_M	ΣW_a
−2	100.9	476.7	4	262.6	1364.1
−1	175.0	898.3	5	234.8	1178.4
0	179.3	903.3	6	148.6	854.6
1	260.5	1170.4	7	167.4	822.4
2	179.7	1137.3	8	192.2	1017.2
3	116.8	632.6			

Chvojkova (1952) plotted two curves showing these variations over many years, for the even and odd 11-year cycles separately. These curves show different cyclic oscillations: the odd-cycle curve attains a maximum once every 80 years, while the even-cycle curve attains one every 55 years. When the two curves are in phase, they have high maxima and deep minima, whereas otherwise the maxima are lower and the minima are shallower. The curves come in phase once every 176 years. Analogous variations were observed by Chvojkova for $2T$ (the duration of the 22-year cycle), T, t, and $\frac{t}{T}$. By extrapolating these cyclic variations to 1957, Chvojkova obtained the following values of W_M for the 19th through 22nd cycles: 100 for cycle 19; 30 for cycle 20; 50 for cycle 21; and 120 for cycle 22. Chvojkova's forecast for the height of the 19th cycle is obviously unsuccessful, and it is quite possible that this is the case for all the other cycles computed by her. As Ol' (1954) has indicated, this is to a considerable extent due to the fact that the cyclic variations observed by Chvojkova are much more complicated, and at the same time much less reliable, than the cyclic variation of W_M^4 over many years which was established by Gleissberg.

§ 6. Eigenson's Method

Eigenson (1955) has proposed a forecasting method based on the properties of Spörer's law. Eigenson and Mandrykina (1954) have shown that there exists a correlation between the average latitude φ_m of the sunspot zone in the epoch of minimum, and the maximum Wolf number W_M for the given cycle. If the 1867 — 1876 cycle is omitted, then the Greenwich data for the sunspot latitudes and the Zurich relative spot numbers give a correlation coefficient $r_{\bar{\varphi}_M, W_M} = +0.73 \pm 0.12$.

The first spots of a new 11-year cycle appear a year or a year and a half prior to the beginning of this cycle as defined by the Schwabe-Wolf law. Therefore, it is possible to predict the height of the next cycle a year or a year and a half before its epoch of minimum. In order to do this, the following regression equation can be used:

$$W_M = 6.4\bar{\varphi}_m - 80.17. \tag{4.15}$$

In 1955 Eigenson used this method to forecast the height of the 19th solar cycle. He used the anomalously high latitude of the first high-latitude one-day group observed at the Mount Wilson Observatory by Babcock on 13 August 1953 (that is, more than a half-year before the epoch of minimum of the 19th cycle). This group was observed simultaneously at the McMath-Hulbert Observatory, and its latitude was +52°.

If the first spot group can be used as a basis for certain, however tentative, conclusions concerning $\bar{\varphi}_m$ and if a correlation coefficient of $r = +0.73$ can be considered high enough for forecasting, then it follows from the foregoing that the 19th solar cycle should be much higher than the 18th. Eigenson used equation (4.15) to obtain a rough estimate of the height of the 19th cycle, namely $W_M \gg 200$. This estimate turned out to be very close to the actually observed maximum Wolf number, the difference being only about 5%. However, this success should be attributed to the extreme care taken by the author in the analysis of his data and to his ability to guess a reasonable lower limit for the figures which he obtained.

Eigenson's method should really be classified as a qualitative method, and in this respect it can only be used in conjunction with methods of ultra-long-range forecasting which give quantitative estimates. The main disadvantage of this method is the obvious arbitrariness of the assumption that the latitude of the first spot group reflects to some extent the average latitude of the spot groups during the epoch of minimum. Moreover, the regression used is very sensitive to changes in $\bar{\varphi}_m$, so that a relatively small change in this latitude can cause a considerable variation in the estimate for the maximum Wolf number. For example, if in the epoch preceding the minimum the latitude $\bar{\varphi}$ of the high-latitude spot groups is 40°, a value which is clearly exaggerated, then we have $W_M = 176$.

§ 7. Schove's Method

The method of ultralong-range Wolf-number forecasting suggested by Schove (1955) is somewhat unique. This method is based on the construction of a series of maximum Wolf numbers, expressed in arbitrary units, and a series of epochs of extrema for the 11-year solar cycles, both series extending from 200 B.C. until 1954. The necessary data were obtained from the available records of sunspots and polar auroras. Fritz has constructed a similar series using the data on hailstorms and high-yield crop years as cycle indicators, but Schove considers the use of such data unjustified. Moreover, he claims that the phase of the polar-aurora cycle lags behind the sunspot cycle somewhat.

Schove bases the compilation of his basic table on two fundamental assumptions:

1) the time between successive maxima is not less than 8 and not more than 16 years;

2) 9 sunspot maxima occur every 100 years.

That this is true has been reliably established for the period from 1515 A.D. until the present. The most reliable data for the extrema of the 11-year solar cycles are those collected after 1749. These data were classified into 30-year groups of longer 11-year cycles (such as those with midpoints in 1650/1655, 1720, 1805/1810, and 1885/1890) and intermediate groups of shorter cycles (such as those with midpoints in 1685, 1760/1765, 1845, and 1935). The average length of seven cycles, at least over the last two and a half centuries, is between 10 and 12 years. The phase of an individual 11-year cycle can easily be determined from the residues which are obtained from the epochs of minimum for the period since 1700, using multiples of 11. During the last three centuries a residue of 5 was typical. The phase of a maximum near the transition from one century to the next, however, is ambiguous.

The main results of Schove's studies are given in Table 14, in which the cycles are listed as decimal fractions. In this century the end years of the solar cycles approximate the following pattern: 00, .11, .22, .33, ..., .88, .99/00 and so they are denoted correspondingly as .0, .1, .2, .3, ..., .8, .0. The intermediate minima are denoted by the decimal fractions .05, .15, .25, ..., .85, .05. In earlier centuries the maxima followed the rule of 11-year periods. Data which cannot be considered reliable are placed in parentheses. When the epoch of minimum was uncertain, the probable error was taken as 4. A probable error of 3 corresponds to cases of ambiguous interpretation.

In order to avoid negative values, the residues in the table were expressed as follows: up to 800 A.D. they ranged from 5 to 15 (instead of from −6 to +4), while after 800 A.D. they ranged from 1 to 13. The intensity of the 11-year cycles was evaluated on the basis of historical sources; an arbitrary qualitative scale was used, with a number of gradations sufficient to characterize the cycle precisely. The intensity symbols have the following meanings:

	Annual Wolf number
SSS = exceptionally strong	>160,
SS = very strong (150, 140, 140)	145,
S = strong (110, 130, 120, 110)	120,
MS = moderately strong (100, 100).	100,
M = moderate (90, 90, 80)	85,
WM = moderately weak (70)	70,
W = weak (60, 60)	60,
WW = very weak (50, 50)	50,
WWW = exceptionally weak	45,
X = unknown	

Here the figures in parentheses correspond to the maximum Wolf numbers for the 11-year cycles since 1750.

TABLE 14

Basic characteristics of solar periodicity for the period from 648 B. C. to 2025 A. D. (according to Schove)

Number of maximum	Year of maximum	Probable error	Residue	Estimated Wolf number	Maximum intensity	Years after previous maximum	Years after previous minimum	Year of minimum	Residue
1	2	3	4	5	6	7	8	9	10
—6.45	—648	3	8		*(S)?*	. . .	. . .	(—653)	3
—5.25	—522	3	12		*(S)?*	. . .	5	(—527)	7
—5.15	(—512)	4	11		. . .	(10)	4	(—516)	7
—5.05	—501	2	11		*(S)?*	(11)	4	(—505)	7
—4.85	(—491)	4	9		*W—M*	(10)	5	(—496)	4
—4.75	—481	2	8		*S*	10	5	—486	3
—4.65	—471	2	7		*S*	10	3	—474	4
—4.55	—461	2	6		*S*	10	4	—465	2
—3.85	—393	3	7		*S*	. . .	6	—397	3
—3.75	. . .	. . .	. . .		. . .	. . .	. . .	—386	3
—3.65	. . .	. . .	. . .		. . .	. . .	. . .	(—375)	. . .
—3.55	. . .	. . .	. . .		. . .	. . .	. . .	(—365)	. . .
—3.45	—349	3	7		*S*	. . .	5	—354	2
—3.35	—340	3	5		*S*	9	4	—344	1
—3.25	. . .	. . .	. . .		. . .	. . .	. . .	—332	2
—2.85	(—293)	4	9		. . .	. . .	. . .	(—298)	2
—2.75	(—283)	4	8		. . .	. . .	. . .	(—288)	1
—2.65	—272	3	6		*X*	. . .	(5)	—277	1
—2.55	(—261)	4	6		. . .	. . .	. . .	—266	1
—2.45	(—249)	4	7		. . .	. . .	. . .	(—254)	2
—2.35	—236	3	9		*X*	. . .	6	—243	3
—2.25	—223	3	11		*X*	13	7	—230	4
—2.15	—214	2	9		*S*	9	5	—219	4
—2.05	—205	2	7		*S*	9	5	—210	2
—1.85	—192	2	2		*S*	13	7	—199	2/1
—1.75	—182	3	7		*WM*	10	5	—187	2
—1.65	—172	3	6		*WM*	10	5	—177	1
—1.55	—163	3	4		*S*	9	4	—167	0
—1.45	(—149)	4	(7)		*W*	(14)	(5)	(—154)	2
—1.35	—135	3	10		*M*	(14)	6	— 141	4
—1.25	—125	3	9		*MS*	10	4	—129	5
—1.15	—113	2	10		*S*	12	6	—119	4
—1.05	—104	2	8		*S*	9	4	—108	4
—0.85	— 91	2	9		*SS*	13	5	— 96	4
—0.75	(— 82)	4	7		*M*	(9)	4	(— 86)	3
—0.65	(— 72)	3	6		*WM*	(10)	5	(— 77)	1
—0.55	— 62	2	5		*SS*	10	7	— 69	—2
—0.45	— 53	2	3		*SS*	9	5	— 58	—2
—0.35	— 42	3	3		*MS*	11	4	— 46	—1
—0.25	— 27	2	7		*S*	15	5	(— 32)	2
—0.15	— 16	3	7		*WM*	11	5	— 21	2
—0.05	(— 5)	4	7		*WM*	(11)	(6)	— 11	1
+0.05	8	4	(8)		*W* or *M*	13	5	(3)	3

TABLE 14 (continued)

1	2	3	4	5	6	7	8	9	10
0.15	20	3	9		*S*	12	5	15	4
0.25	(31)	4	(9)		*W* or *M*	(11)	(5)	26	4
0.35	42	3	9		*W* or *M*	(11)	5	37	4
0.45	53	3	9		*S*	11	6	47	3
0.55	65	3	10		*S*	12	5	60	5
0.65	(76)	4	(10)		*W* or *M*	(11)	6	(70)	4
0.75	(86)	4	(9)		*W* or *M*	(10)	6	(80)	3
0.85	(96)	4	(8)		*W* or *M*	(10)	5	(91)	3
1.05	105	3	5		*M* or *S*	9	4	101	1
1.15	(118)	4	7		*W* or *M*	(13)	6	112	1
1.25	(130)	4	8		*W* or *M*	(12)	6	(124)	2
1.35	(141)	4	8		*W* or *M*	(11)	6	(135)	2
1.45	(152)	4	8		*W* or *M*	(11)	6	(146)	2
1.55	(163)	4	8		*W* or *M*	(11)	6	(157)	2
1.65	175	3	9		*M* or *S*	(12)	5	170	4
1.75	186	3	9		*S*	11	4	182	5
1.85	196	3	8		*S*	10	4	192	4
2.05	(208)	4	8		*X*	(12)	5	203	3
2.15	(219)	4	8		*X*	(11)	5	(214)	3
2.25	(230)	4	8		*X*	(11)	5	(225)	3
2.35	(240)	4	7		*X*	(10)	5	(235)	2
2.45	(252)	4	8		*X*	(12)	5	(247)	3
2.55	(265)	4	10		*X*	(13)	5	(260)	5
2.65	(277)	4	11		*X*	(12)	5	(272)	6
2.75	290	3	13		*M* or *S*	13	6	(284)	7
2.85	302	1	14/13		*SS*	12	6	296	8
3.05	311	1	11		*M*	9	4	307	7
3.15	321	1	10		*M*	10	4	317	6
3.25	330	3	8		*W*	9	4	326	4
3.35	342	2	9		*W*	12	6	336	3
3.45	354	2	10		*S*	12	6	348	4
3.55	362	3	7		*S*	8	4	358	3
3.65	372	1	6		*SS*	10	4	368	2
3.75	387	3	10		*M*	15	7	380	3
3.85	396	3	8		*M*	9	5	391	3
4.05	410	4	10		*W*	14	6	404	4
4.15	(421)	4	(10)		*W*	11	5	416	5
4.25	430	1	8		*M* or *S*	9	4	426	4
4.35	441	1	8		*M* or *S*	11	4	437	4
4.45	452	2	8		*S*	11	4	448	4
4.55	(465)	4	(10)		*W*	(13)	6	459	4
4.65	479	1	13		*M* or *S*	(14)	7	472	6
4.75	490	1	13		*M*	11	6	484	7
4.85	501	1	13/12		*SS*	11	6	495	7
5.05	511	1	11		*S*	10	4	507	7
5.15	522	2	11		*W* or *M*	11	5	517	6
5.25	531	1	9		*S*	9	5	526	4
5.35	542	3	9		*M*	11	4	538	5
5.45	557	1	13		*M*	15	6	551	7
5.55	567	1	12		*SS*	10	5	562	7
5.65	578	2	12		*M*	11	5	573	7

TABLE 14 (continued)

1	2	3	4	5	6	7	8	9	10
5.75	585	3	8		*S*	7	3	582	5
5.85	597	2	9		*W* or *M*	12	5	592	4
6.05	(607)	4	(7)		*W*	10	5	(602)	2
6.15	618	2	7		*M*	11	5	613	2
6.25	628	3	6		*W—M*	10	5	623	1
6.35	642	2	9		*M*	14	5	637	4
6.45	654	2	10		*M—S*	12	5	649	5
6.55	665	2	10		*M*	11	5	660	6
6.65	677	2	10		*S*	12	6	671	5
6.75	(689)	4	13		*W*	12	5	684	7
6.85	(699)	4	12		*W*	10	6	(693)	5
7.05	714	1	14		*S*	15	7	(707)	7
7.15	724	1	13		*S*	10	5	719	8
7.25	735	3	13		*W*	11	5	730	8
7.35	745	1	12		*SS*	10	6	739	6
7.45	754	2	10		*M*	9	5	(749)	5
7.55	765	1	10		*SS*	11	4	761	6
7.65	(776)	4	10		*MS*	(11)	6	(770)	4
7.75	(787)	4	10		*W*	(11)	5	(782)	5
7.85	(798)	4	10		*MS*	(11)	5	(793)	5
8.05	809	1	9		*S*	11	5	804	4
8.15	821	3	10		*W*	12	6	815	4
8.25	829	2	8		*S*	8	4	825	4
8.35	840	1	7		*SS*	11	4	836	3
8.45	850	1	6		*MS*	10	4	846	2
8.55	862	1	7		*M*	12	6	856	1
8.65	872	3	6		*S*	10	4	868	2
8.75	887	3	10		*M*	15	5	882	5
8.85	898	3	10		*W*	11	5	893	5
9.05	907	2	7		*W*	8	5	902	2
9.15	917	2	6		*M*	11	5	912	1
9.25	926	1	4		*SS*	9	5	921	—1
9.35	938	2	5		*MS*	12	4	934	1
9.45	(950)	3	6		*WM*	12	5	(945)	1
9.55	963	2	8		*SS*	13	4	959	4
9.65	974	3	8		*SS*	11	4	970	4
9.75	986	2	9		*M*	12	4	982	5
9.85	(994)	2	6		*W*	8	(4)	(990)	2
10.05	1003	1	3		*S*	9	(5)	(998)	—1/—2
10.15	1016	1	5		*M*	13	5	1010	—1
10.25	1027	1	5		*M*	11	5	1022	0
10.35	1038	3	5		*W*	11	4	1034	+1
10.45	(1052)	4	8		*WW*	14	5	(1047)	3
10.55	1067	4	12		*M*	15	7	(1060)	5
10.65	1078	2	12		*M*	11	7	1071	5
10.75	1088	1	11		*M*	10	6	1082	5
10.85	1098	1	10		*SS*	10	6	1092	4
11.05	(1110)	3	10		*WM*	12	4	1106	6
11.15	1118	1	7		*SS*	8	3	1115	4
11.25	1129	1	7		*S*	11	5	1124	2
11.35	1138	1	5		*SS*	9	4	1134	1

TABLE 14 (continued)

1	2	3	4	5	6	7	8	9	10
11.45	1151	1	7		*S*	13	6	1145	1
11.55	1160	2	4		*WM*	9	5	1155	0
11.65	1173	1	7		*MS*	13	6	1167	1
11.75	1185	1	8		*MS*	12	5	1180	3
11.85	1193	1	5		*M*	8	3	1190	2
12.05	1202	1	2		*SS*	9	3	1199	0/—1
12.15	1219	3	8		*M*	17	7	1212	1
12.25	1228	1	6		*M*	9	4	1224	2
12.35	1239	3	6		*M*	11	6	1233	0
12.45	1249	3	5		*WM*	10	5	1244	0
12.55	1259	2	4		*M*	10	3	1256	1
12.65	1276	3	10		*M*	17	7	(1269)	3
12.75	1288	1	11		*M*	12	6	1282	5
12.85	1296	3	8		*W*	8	5	(1291)	3
13.05	1308	2	8		*M*	12	7	1301	1
13.15	1316	2	5		*M*	8	5	1311	0
13.25	1324	2	2		*M*	8	5	1319	—3
13.35	1337	3	4		*W*	13	5	1332	—1
13.45	1353	2	9		*WM*	16	7	1346	+2
13.55	1362	2	7		*SS*	9	4	1358	3
13.65	1372	1	6		*SSS*	10	4	1368	2
13.75	1382	2	5		*MS*	10	4	1378	1
13.85	1391	3	3		*M*	9	5	1386	—2
14.05	1402	1	2		*M*	11	6	1396	—3/—4
14.15	(1413)	3	2		*WW*	11	6	1407	—4
14.25	(1429)	4	7		*WM*	16	8	(1421)	—1
14.35	(1439)	3	6		*WM*	10	5	1434	+1
14.45	1449	4	5		*WM*	10	6	1443	—1
14.55	1461	3	6		*WM*	12	4	1457	2
14.65	(1472)	4	6		*WW*	11	4	1468	2
14.75	(1480)	4	3		*WW*	8	4	(1476)	—1
14.85	1492	4	4		*WM*	12	4	(1488)	0
15.05	1505	3	5	(60)	*W*	13	7	1498	—1/—2
15.15	1519	1	8	(80)	*M*	14	7	1512	+1
15.25	1528	1	6	(150)	*SS*	9	3	1525	+3
15.35	1539	1	6	(130)	*S*	11	4	1535	+2
15.45	1548	1	4	(120)	*S*	9	5	1543	—1
15.55	1558	1	3	(160)	*SS*	10	5	1553	—2
15.65	1572	1	6	(150)	*SS*	14	5	1567	+1
15.75	1581	1	4	(130)	*S*	9	3	1578	+1
15.85	1591	2	3	(70)	*WM*	10	4	1587	—1
16.05	1604.5	2	4	(80)	*WM*	13	5	1599.5	0/—1
16.15	1615.5	1	4	(90)	*M*	11	4.7	1610.8	—0.7
16.25	1626.0	1	3.5	(100)	*MS*	10.5	7	1619.0	—3.5
16.35	1639.5	1	6	(70)	*WM*	13.5	5.5	1634.0	+0.5
16.45	1649.0	1	4.5	(40)	*WW*	9.5	4	1645.0	+0.5
16.55	1660.0	1	4.5	(50)	*WW*	11.0	5	1655.0	—0.5
16.65	1675.0	1	8.5	(60)	*W*	15.0	9	1666.0	—0.5
16.75	1685.0	1	7.5	(50)	*WW*	10.0	5.5	1679.5	+2
16.85	1693.0	1	4.5	(30)	*WWW*	8.0	3.5	1689.5	+1
17.05	1705.5	1	5.0	(50)	*WW*	12.5	7.5	1698.0	—1.5/—2.5

TABLE 14 (continued)

1	2	3	4	5	6	7	8	9	10
17.15	1718.2	1	6.7	(130)	*S*	12.7	6.2	1712.0	+0.5
17.25	1727.5	1	5.0	(140)	*SS*	9.3	4.0	1723.5	+1.0
17.35	1738.7	1	5.2	(110)	*S*	11.2	4.7	1734.0	+0.5
17.45	1750.3	. . .	5.8	80	*M*	11.6	5.3	1745.0	+0.5
17.55	1761.5	. . .	6.0	90	*M*	11.2	6.3	1755.2	−0.3
17.65	1769.7	. . .	3.2	110	*S*	8.2	3.2	1766.5	0
17.75	1778.4	. . .	+0.9	150	*SS*	8.7	2.9	1775.5	−2.0
17.85	1788.1	. . .	−0.4	130	*S*	9.7	3.4	1784.7	−3.8
18.05	1805.2	. . .	+4.7	50	*WW*	17.1	6.9	1798.3	−1.2/−2.2
18.15	1816.4	. . .	4.9	50	*WW*	11.2	5.8	1810.6	−0.9
18.25	1829.9	. . .	7.4	70	*WM*	13.5	6.6	1823.3	−0.8
18.35	1837.2	. . .	3.7	140	*SS*	7.3	3.3	1833.9	+0.4
18.45	1848.1	. . .	3.6	120	*S*	10.9	4.6	1843.5	−1.0
18.55	1860.1	. . .	4.6	100	*MS*	12.0	4.1	1856.0	+0.5
18.65	1870.6	. . .	4.1	140	*SS*	10.5	3.4	1867.2	+0.7
18.75	1883.9	. . .	6.4	60	*W*	13.3	5.0	1878.9	+1.4
18.85	1894.1	. . .	5.6	90	*M*	10.2	4.5	1889.6	+1.1
19.05	1907.0	. . .	6.5	60	*W*	12.9	5.3	1901.7	+1.2
19.15	1917.6	. . .	6.1	100	*MS*	10.6	4.0	1913.6	+2.1
19.25	1928.4	. . .	5.9	80	*WM*	10.8	4.8	1923.6	+1.1
19.35	1937.4	. . .	2.9	110	*S*	9.0	3.6	1933.8	+0.3
19.45	1947.5	. . .	3.0	150	*SS*	10.1	3.3	1944.2	−0.2
19.55	(1958.5)	2	(3)	. . .	*SS*	. . .	(4)	(1954.5)	(−1)
19.65	(1972.5)	2	(6)	. . .	*M/S*	. . .	(6)	(1966.5)	(0)
19.75	(1984.5)	2	(7)	. . .	*SS*	. . .	(6)	(1978.5)	(+1)
19.85	(1994.5)	2	(6)	. . .	*S*	. . .	(5)	(1989.5)	(+1)
20.05	(2004.5)	2	(5/4)	. . .	*M/S*	. . .	(4)	(2000.5)	(+1/0)
20.15	(2014.5)	2	(3)	. . .	*M*	. . .	(5)	2009.5)	(−2)
20.25	(2025.5)	2	(4)	. . .	. . .	. . .	. . .	(. . .	. . .

An examination of the data for the period since 1610 shows that the minimum generally precedes the maximum by 4 years if the maximum is strong or very strong, by 5 years if the maximum is moderate or moderately strong, and by 6 years if the maximum is weak. The intervals of time elapsing from minimum to maximum, for the period since 1850, satisfy the formula

$$t = 7 - 0.03W_M, \tag{4.16}$$

where W_M is the yearly maximum Wolf number. The minimum following a weak maximum is generally separated from it by about 6 years, while the minimum following a strong maximum is separated from it by about 7 years.

It is interesting that the table also implies the existence of the 80-year to 90-year cycle, not only for the period considered by Gleissberg but also for a much earlier time. The sunspot cycle is longer in weak (according to the polar auroras) periods and shorter in active periods.

The average sunspot cycle over about 500 years ranges between 11.03 and 11.14 years. Since 200 B.C. there have been an average of 90 to 91 cycles per millennium. The positions of the minima of the 11-year cycles are fairly well approximated by the arithmetic progression 1932 − 155.2 n, where n is the cycle number. This formula makes it possible to compute the epochs of minima which are not given by observational data. In addition to the 80-year to 90-year cycle, the variations in the lengths of the 11-year cycles since 1510 can be represented quite well by a long 160-year to 170-year cycle. The average cycle during the periods 1560-1590, 1750-1790, and 1900-1950 was 10 years long, while that during the periods

1600—1670 and 1780—1820 was 12 years long. This alternation in cycle length follows indirectly from the 200-year cycle of polar-aurora activity.

Periods of increased auroral activity in even centuries are characterized by two maxima with a short quiet period in between. The midpoints of the maxima about the short quiet periods are as follows: 1755, 1555, 1350, 1160, 955, 755, 540, 340, ..., the period between the two basic peaks being about 50 years.

The average behavior of the 11-year sunspot cycles starting with 900 A.D. may be expressed as follows:

Year	Residue	Intensity	Years after preceding maximum
07	(7)	***W***	...
17	(6)	***S***	10
27	(5)	***S***	10
38	(5)	***S***	11
50	(6)	***M***	12
61	(6)	***S***	11
72	(6)	***SS***	11
82	(5)	***S***	11
91	(3)	***WM***	8
03	(4/3)	***WM***	12

This table can, provided the 80-year to 90-year cycle is taken into account, be used as a key for the prediction of future 11-year cycles. Schove has also discovered evidence for cycles with durations of over 200 years, in particular a 554-year cycle, but he does not claim that these cycles actually exist. It should be noted that the 554-year intervals occur between the highest-intensity maxima of solar activity.

Thus, Schove's method for ultralong-range forecasting is suitable for even centuries only and does not take into account cycles longer than 80 or 90 years. Schove used his method to forecast the main characteristics for cycles 19 through 25. It is true that for the 19th cycle Schove followed Gleissberg (1944b) in assuming that this cycle should be very strong, so that he dated the central maximum at 1970 rather than 1960. However, with all its shortcomings, Schove's method is of unquestionable interest simply because of the vast amount of data on which it is based. There is, of course, a certain amount of uncertainty involved in the historical data used, but this still does not justify a total rejection of these data, especially after the very careful analysis made by Schove.

§ 8. Forecasts of Sunspot Areas

So far we have just considered the forecasting of Wolf numbers. Until very recently, no methods for forecasting the sunspot areas had been developed, and the first steps in this direction were taken only in the last few years. Just as for the Wolf numbers during the first years of developing prediction methods, most of the attempts made to develop methods have been directed toward ultralong-range forecasting of sunspot areas.

It should be noted first that, since the data available for the spot-group areas only cover seven complete 11-year cycles, most of the properties described should be considered as purely qualitative. In this case, therefore, high accuracy of the forecasts is not to be expected.

Two methods have been worked out for spot-group-area prediction, that of Xanthakis (1959) and that of Bezrukova (1958). Since the method of Xanthakis has never been applied in practice, except for back calculations over previous cycles, therefore it will not be considered in detail here and it will only be discussed briefly at the end of the section.

Bezrukova's method is based on the asymmetry of sunspot activity in the northern and southern solar hemispheres, a factor which was discussed in detail in Chapter I, § 9. In practice, for the prediction of all the characteristics of the 11-year solar cycle, this method is applicable only to odd cycles (Zurich system). This method will now be considered using the forecast of spot areas for the 19th cycle as an example.

Let us recall that, according to Bezrukova, during any given cycle the cyclical curve is single-maximum in one of the solar hemispheres and deformed (or double-maximum) in the other. Therefore, the following characteristics of the 11-year cycle for the spot-area index can be determined: the height of the single maximum and the height of the corresponding point on the double-maximum curve. In order to determine the sunspot-group area in the year of maximum of the single-maximum cycle, let us make use of the correlation between the spot areas in the year of the first maximum of a double-maximum even cycle and the spot area in the year of maximum of a single-maximum odd cycle. Table 15 gives the corresponding data for cycles 12 through 18.

TABLE 15

Spot areas for solar hemispheres in the epochs of extrema of the 11-year cycles (according to Bezrukova)

Cycle number	First maximum of double-maximum cycle	Maximum of single-maximum cycle	Ratio
12	500	815	1.882
13	607	941	
14	400	750	1.977
15	318	860	
16	663	679	1.986
17	678	1317	
18	1127	1645	
19		(2250)	

The table indicates that, for all three pairs of cycles designated in the fourth column of the table, the spot area of the single-maximum odd cycle is almost twice as great as the spot area of the first maximum of the preceding double-maximum even cycle. If we take into account the gradual increase of this ratio, then we see that for the 19th cycle the spot-group area in the year of maximum of the single-maximum cycle can be expected to be 2250 millionths of a solar hemisphere (m.s.h.). If we take the second differences into account, we obtain 2297 m.s.h.

We next plot the spot area of the single-maximum cycle in the year of maximum as a function of the spot area in the year of the first maximum of the double-maximum cycle. The graph shows that the relationship is linear for the three pairs of cycles. If on the straight line we read off the point corresponding to 1127 m.s.h., then we obtain 1820 m.s.h. Then, by taking the average of the two values found for the spot area in the year of maximum of the single-maximum cycle, we obtain 2050 m.s.h.

In order to find the spot area of the double-maximum cycle in the year of maximum of the single-maximum cycle, let us refer to Table 16. This table shows that the ratio between the spot area of the double-maximum cycle and that of the single-maximum cycle in the year of maximum of the single-maximum cycle is variable. This ratio apparently follows a 44-year cycle. If we take this factor into account, then we can determine the spot area of the double-maximum cycle in the year of maximum of the single-maximum cycle from the previously determined spot areas in this year. The numbers in parentheses for the 19th cycle were obtained using spot areas of 2297 m.s.h. and 2050 m.s.h., respectively, for the single-maximum cycle.

TABLE 16

Spot areas for solar hemispheres in the year of maximum of the single-maximum cycle (according to Bezrukova)

Cycle number	Year	Single-maximum cycle	Double-maximum cycle	Ratio	Difference
12	1883	340	815	2.379	
13	1893	517	941	1.820	
14	1905	440	750	1.704	
15	1917	677	860	1.270	
16	1927	379	679	1.791	
17	1937	757	1317	1.739	0.052
18	1947	992	1645	1.658	0.081
19	1957	(1440) (1290)		(1.592)	

For the two remaining characteristics of the double-maximum cycle, Bezrukova's method gives only a very approximate estimate. An analysis of the data shows that the area of the first maximum of the odd double-maximum cycle in every pair of cycles does not exceed the area of the preceding even single-maximum cycle in its year of maximum. On the basis of this, it may be assumed that for the first maximum of the 19th double-maximum cycle this area does not exceed 1645 m.s.h. On the other hand, the behavior of the Hale pairs of cycles implies that this area should be greater than the spot area of the previous cycle, which was 1127 m.s.h. Moreover, this area is definitely greater than the spot area of the double-maximum cycle in the year of maximum of the single-maximum cycle, that is, it is greater than 1290 m.s.h. This also applies to the area of the second maximum of the 19th double-maximum cycle.

Bezrukova's forecast for the 19th cycle is a fairly accurate one, and this justifies the use of her method for the prediction of spot areas. The main defects of the method are its limited applicability (it applies to odd 11-year cycles only) and the semiqualitative nature of the estimates obtained.

The method of Xanthakis (1959) is based on the relationship between the principal characteristics of the 11-year cycle and the length of its rising part. Let us introduce the following quantities: S_N and S_S, the maximum spot areas in the northern and southern solar hemispheres; N, the cycle number; and t_N and t_S, the lengths of the rising part of the cycle in the northern and southern hemispheres.

If t_N and t_S are expressed in solar rotations, then, according to Xanthakis, the maximum spot areas (per rotation) in the northern and southern hemispheres can be computed from the expressions

$$S_N = 1056 + 2.6(t_N - 65)^2 + 730 \sin (N - 5)\frac{2\pi}{8}, \tag{4.17}$$

$$S_S = 1280 + 5.8(t_S - 62)^2 - 580 \sin (N - 5)\frac{2\pi}{8}. \tag{4.18}$$

For the entire solar disk, the average monthly maximum spot area is

$$S = 2171 + 2.37(t - 65)^2, \tag{4.19}$$

where t is in months. To calculate the yearly maximum sunspot area, we use the formula

$$S = 1060 + 1.65(t - 67)^2, \tag{4.20}$$

where t is in months, or the formula

$$S = 1100 + 240(t - 5.6)^2, \tag{4.21}$$

where t is in years.

The method of Xanthakis gives quite satisfactory results in back calculations, but for forecasts its accuracy is apparently much lower, since the prediction of the length of the rising part of the cycle is very complicated. Xanthakis did not concern himself with this problem at all, although it is of primary significance for his method. Actually, the variation of the length of the rising part of the 11-year cycle for the spot areas is so complex in nature that is impossible to refer, with any degree of certainty, to anything other than a purely qualitative estimate. Thus, the quantitative value of Xanthakis's method is practically negligible, and the method is of academic interest only. In this aspect it is similar to Schuster's method (see § 2 of this chapter), which served more or less as a point of departure for the development of other methods of ultralong-range forecasting of Wolf numbers, these other methods being useful for practical forecasts.

It seems to us that, due to the obviously insufficient volume of available data, no reliable method for the ultralong-range forecasting of spot-group areas can be developed at present. Much more success is possible, however, using methods for the prediction of this index within the current 11-year cycle. Therefore, special attention should now be given to the study of the intracycle regularities in the variation of sunspot-group areas.

§ 9. Concluding Remarks

Ultralong-range forecasts of solar activity, in spite of their long history, still do not give satisfactory results. Different methods often lead to diametrically opposite conclusions, so that the difficulty is further increased by the fact that we cannot take averages of the results obtained by the various methods without leading to error. Suffice it to say that the forecasts offered by different authors for the 19th cycle ranged from 50 to 208. This wide range of predicted heights for the 19th cycle is due more to fundamental differences in the initial assumptions of the various authors than to an inherently low accuracy of the ultralong-range forecasts. Therefore, we are forced simply to prefer some particular forecast mainly on the strength of its basic premises.

Of all the methods for the ultralong-range forecasting of Wolf numbers discussed in this chapter, special attention should be given to the methods of Gleissberg and Ol', and especially to the latter since it provides a means of internal control. Finally, let us note that, in view of the preceding considerations, any evaluation of the forecast accuracy in this case is meaningless.

Chapter V

FORECASTS OF THE BASIC CHARACTERISTICS OF THE 20TH CYCLE

§ 1. General Considerations

Forecasts of the basic characteristics of the 20th solar cycle are especially difficult, chiefly because, according to most investigators, the 19th sunspot cycle represents the extremum of the 80-year to 90-year cycle and is distinguished by an exceptionally high intensity. In any case, no 11-year cycles with comparably high intensities have been recorded during the entire period of telescopic observations of the sun. Almost everyone agrees that the next (the 20th) cycle will definitely be lower than the current (the 19th) cycle. However, what we wish to determine is just how much lower it will be, and this is the main problem.

So far, when giving forecasts for the cycle to come, it has been sufficient merely to take into account the properties of all the cycles recorded during the period of telescopic solar observations. The 19th cycle, however, was the first extraordinary cycle to be observed, and the predictions made for it have shown that certain additional, still unknown, factors must be taken into consideration. Nevertheless, the 19th cycle, which was odd, was the second cycle of a Hale pair, and this simplifies the problem to a certain extent. But now we are faced with a dilemma, namely whether the drop in activity from the present cycle to the next cycle will be exceptionally large or whether it will remain almost equal to the previous drop (that is, not more than 80 or 90, in terms of Wolf numbers). This uncertainty has led investigators to seek new methods of ultralong-range forecasting, and in some cases to seek even purely qualitative methods. Up to the present very few works have been published on this subject, but these will be discussed in detail in this chapter.

The only exception to the prevailing opinions for the predicted trend of solar activity is that of Chadwick (1959) and his viewpoint is as follows. Since it is equally certain that 178-year and 169-year sunspot cycles exist, this author maintains that there is equal certainty that the next (20th) cycle will be either higher or lower than the 19th cycle. Now, this proposition actually contibutes nothing outside of its skepticism, and in fact it is also equally certain that the 178-year and 169-year sunspot cycles are purely hypothetical. This is the reason, by the way, why these cycles are not taken into consideration in any of the forecasts for the 20th cycle. Consequently, Chadwick's viewpoint will be disregarded in the following.

In addition, a new element which has only recently appeared in connection with forecasting the basic characteristics of the 20th cycle is the development of an alternative approach to the prediction of the epochs of extrema for the 11-year cycles. This new approach is largely due to the fact that the regressions of Waldmeier (1935), which correlate the temporal characteristics of the 11-year cycle with its intensity, now give a much lower accuracy than when they were originally applied. This may be caused by the existence of some "long" cycle of solar activity which so far has escaped notice (see Chapter I, § 11).

Finally, the various difficulties involved have led many authors to give forecasts for only some (or even for just one) of the characteristics of the 20th solar cycle and to confine themselves to individual comments concerning the other characteristics or simply to neglect them entirely.

§ 2. Bezrukova's Forecast

The forecast of Bezrukova (1959b) gives only the heights of the 20th and 21st sunspot cycles and is semiqualitative in nature. The method used is somewhat reminiscent of that of Chvojkova (see Chapter IV, § 5) but is considerably simpler than the latter.

Bezrukova considers separately the variations in height of odd and even 11-year cycles. She observes that for odd cycles this variation is more regular than for even cycles, and that for odd cycles there is an alternation of the heights of many-year cycles made up of odd cycles. As shown by Figure 15, many-year cycle I is high, cycle II is low, and cycle III is

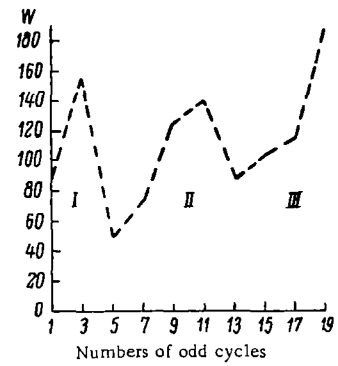

FIGURE 15

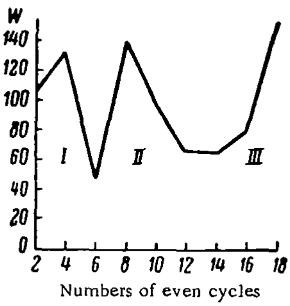

FIGURE 16

once again high. In addition, the high maximum of cycle I is followed by a low minimum in the fifth 11-year cycle, while the low maximum of cycle II is followed by a high minimum in the 13th cycle. It is likely that the high maximum of cycle III will be followed by a low minimum in the 21st 11-year cycle, that is, that this cycle will be definitely lower than the 13th cycle, whose maximum Wolf number was 84.9. If we assume that, as indicated by Figure 15, the height of the minima of the many-year cycles continually increases from the 5th through the 13th to the 21st cycle, then W_M for the odd 21st cycle should be approximately 120.

The curve for the variation over many years of the height of the even 11-year cycles (Figure 16) also shows a gradual upward change in the height of these cycles. If we assume that in this case as well the height of the minima of the many-year cycles continually increases, then W_M for the even 20th cycle may be about 75. If, however, these heights alternate in pairs, then the 20th cycle will be lower than the 14th cycle, the maximum Wolf number of which was 63.5.

Thus, according to Bezrukova's forecast, the maximum Wolf number of the 20th sunspot cycle will be less than 64 or else about 75, while that of the 21st cycle will be less than 85 or about 120. Earlier, Bezrukova (1959a) had found from an analysis of the curve for the variation of sunspot activity over many years that the maximum Wolf number of the 20th solar cycle could be from 45 to 85. Her later forecast (1959b) gives a somewhat narrower range of Wolf numbers, so that a separation of the cycles into even and odd improves the forecast appreciably.

Despite the qualitative nature of Bezrukova's forecast, it appears to be more reliable than that of Chvojkova ($W_M = 30$ for the 20th cycle), which is based on approximately the same premises as Bezrukova's method but which involves a quite complicated mathematical apparatus.

Bezrukova does not give the epochs of extrema of the 20th and 21st sunspot cycles, since her method does not enable a prediction of these characteristics of the 11-year cycle. This is obviously one of the main defects of this method of forecasting. Moreover, Bezrukova's method actually assumes that the behavior of the 80-year to 90-year sunspot cycles will not change in the future. That this is true seems to us to be very unlikely, and consequently Bezrukova's forecast for the 20th cycle may well be too low. The advantages of Bezrukova's method are its simplicity and the fact that it utilizes the properties of the 22-year sunspot cycle, properties which have a definite physical significance.

§ 3. Minnis's Forecast

Forecasting the height of the 20th sunspot cycle has been considered in more detail by Minnis (1960), who takes as his initial data a series of twenty maximum smoothed monthly Wolf numbers. He maintains that to establish the height limits for the next (20th) sunspot cycle it is necessary to estimate objectively whether this minimum will lie between certain specified limits. In order to ensure higher reliability, Minnis applies three different methods, employing 1) a direct sequence, 2) the frequency distribution of ΔW_M and W_M, and 3) an autocorrelation function. Let us now consider each of these methods separately.

The differences ΔW_M between the maximum relative spot numbers can be either positive or negative. Let us introduce the following notation: if a given difference in the series has the same sign as the preceding one, then this trend in the variation of successive maxima will be designated as s; if two successive differences have opposite signs, then the trend will be designated as d. By applying this notation to the variations in height of the maxima of the 11-year cycles, we obtain the following direct sequence, which may also be easily deduced from Figure 1:

$$dsdssdsdsdddddddss$$

We wish now to evaluate the relative probability that the next term in this sequence will be s (in which case the next cycle will be higher than the current cycle) or d (in which case the next cycle will be lower than the current cycle).

The heights of the 11-year solar cycles since 1750 have ranged on the average between 50 and 150, with the exception of the 19th cycle. Since the average and most probable values of ΔW_M are about 35, therefore it can be expected that the probability of two or more successive changes in the same direction will be lower than the probability of a sequence of changes in the opposite direction. Thus the probability p_1 that s will occur is lower than the probability $(1-p_1)$ that d will occur. This hypothesis is actually verified by the frequencies of occurrence for s and d, which give $p_1 = 0.4$. Consequently, it follows that the probabilities p_2 and p_3 that a second and a third s will occur after one or two changes in the same direction are less than 0.4. Since $p_1 > p_2$, we obtain the best agreement between the computed and observed frequencies of occurrence of s and d by setting $p_1 = 0.4$, $p_2 = 0.3$ or 0.35, and $p_3 = 0.3$.

Using these values for p, Minnis calculated the expected number of occurrences for four possible combinations of s and d. Moreover, these numbers were calculated assuming that s and d occur with equal probability under all conditions. The results are given in Table 17.

TABLE 17

Expected number of occurrences for four possible combinations of d and s (according to Minnis)

	Probability			Number of occurrences				χ^2
	p_1	p_2	p_3	dd	ds	sd	ss	
Calculated (a)	0.5	0.5	0.5	4.3	4.3	4.2	4.2	2.06
Calculated (b)	0.4	0.35	0.3	6.3	4.2	4.3	2.1	0.19
Calculated (c)	0.4	0.3	0.3	6.5	4.4	4.3	1.8	0.17
Observed . . .	—	—	—	6	5	4	2	

The table shows that, according to the χ^2 criterion, the fit obtained in cases (b) and (c) is essentially the same as that in case (a). Nevertheless, cases (b) and (c) appear more plausible on physical grounds and also because the probability of occurrence of seven successive d-terms, a combination which is observed in the direct sequence, would have been less than 0.01 for $P = 0.5$.

Thus it was decided that the best estimates are $p_3 = 0.3$ and $(1-p_3) = 0.7$. But since the direct sequence up to 1958 (see Figure 1) terminates in ss, therefore the probability that the next cycle will be higher than the present one is 0.3 and the probability that it will be lower is 0.7.

Let us now consider the frequency distribution of ΔW_M. The curve has two maxima, and the mean and minimum frequencies of ΔW_M lie near $\Delta W_M = 0$. Neglecting signs, we obtain an average value for $/\Delta W_M/$ of 35, with a standard deviation of 23. Consequently, for a negative ΔW_M between the maxima of the 19th and 20th cycles the estimated value of W_M for the 20th cycle is 168 ± 23, while for a positive ΔW_M it is 238 ± 23.

It is also useful to note that ΔW_M was greater than 75 in only one out of nineteen cases and that it has never been greater than 92. A subtraction for the current (19th) cycle shows that in the next (20th) cycle we may expect $W_M > 128$ with a probability of 0.95 and $W_M > 111$ with an even higher probability. Analogous arguments based on the distribution of W_M lead to the conclusion that the probability of $W_M < 159$ in the 20th cycle is 0.95.

The next method used by Minnis consists in the use of an autocorrelation function computed from a series of twenty values of W_M. The form of this function shows that periodicity apparently exists, the period length being seven or eight 11-year cycles. However, except for the cases $r=1$ and $r=3$ (where r is the distance between the correlated values of W_M, expressed in cycles), the autocorrelation coefficients are not appreciably different from zero.

The author next applies the ($W_{M,n}$; $W_{M,n+r}$) regression equations for $r=1$ and $r=3$ to forecast the height of the next (20th) cycle. The values obtained for the maximum smoothed monthly relative spot numbers are 154 ±38 and 97 ±36, respectively. It should be noted that even in these cases the autocorrelation coefficients are low, while the standard deviations of the estimates obtained are too high. Therefore, it can hardly be expected that this method will give forecasts of high accuracy.

In order to obtain the most reliable results, Minnis combines all his estimates of the height of the 20th sunspot cycle, these being listed in Table 18.

TABLE 18

Estimates of height of 20th sunspot cycle (according to Minnis)

	Method used	W_M	Probability
1	Direct sequence	> 203	0.3
2	Direct sequence	< 203	0.7
3	Distribution of W_M	< 159	0.95
4	Distribution of ΔW_M.	> 128	0.95
5	Distribution of ΔW_M.	> 111	> 0.95
6	Distribution of ΔW_M.	238±23	0.68
7	Distribution of ΔW_M.	168±23	0.68
8	Autocorrelation ($r=1$)	154±38	0.68
9	Autocorrelation ($r=3$)	97±36	0.68

The most objective means of combining these individual estimates would apparently be to calculate the average of estimates 6 through 9 and to weight each estimate in inverse proportion to its variability [standard deviation]. Moreover, the weights of estimates 6 and 7 should be reduced to 0.3 and 0.7, respectively, in order to make them compatible with estimates 1 and 2, so that their combined weight is unity, just as is the [combined] weight of estimates 8 and 9. If this procedure is followed, then, with a probability of 0.68, the maximum smoothed monthly Wolf number for the 20th solar cycle should lie between 104 and 218.

However, this interval is very wide, mainly because it was obtained using estimates whose probabilities are definitely negligible. Actually, there is a probability of 0.3 that the value of ΔW_M between the maxima of the 19th and 20th cycles will be positive. Thus it follows that the probability that $W_M > 159$ is only 0.05 (estimate 3). If we assume that these two calculations are independent, then their combined probability is only 0.02. Consequently, in practice we may reject the possibility that the 20th cycle will be higher than the 19th cycle. Thus estimate 6 can be dropped, and the subsequent discussion can be confined to estimates 3, 4, 5, 7, 8, and 9 only.

The weighted mean for estimates 7 through 9 is 149, with a standard deviation of 45. Consequently, the probability that in the 20th cycle W_M will lie between 104 and 194 is 0.68. Estimates 3 and 5 taken together lead to the conclusion that the height of the next (20th) cycle will lie between 111 and 159, with a probability of 0.9. Both of these estimates show that the maximum Wolf number for the 20th cycle will apparently be greater than 108, which is the average height of the last twenty sunspot cycles for the years 1750 through 1950.

The final limits for the height of the 20th cycle which were adopted by Minnis are 110 to 160, with a probability of 0.75. Minnis's forecast is actually based on the variation of solar activity during the last twenty cycles, and thus it has the same disadvantages as Bezrukova's forecast. However, in contrast to Bezrukova, Minnis uses quantitative methods. On the other hand, the probabilities that he gives do not actually characterize the reliability of any of his estimates. In this respect, the methods used by Minnis are to some extent similar to Gleissberg's method. Finally, Minnis attaches too much significance to the average values of ΔW_M, although, as shown by Figure 1, exceptionally high 11-year cycles are followed by a very sharp drop in sunspot activity.

Minnis arbitrarily gives the epoch of maximum of the 20th solar cycle as 1968. However, he does not deal especially with this subject and apparently gives this year only because during recent 11-year sunspot cycles the epochs of maximum have occurred at intervals of approximately 10 years. The year given by Minnis should thus be regarded as just a synonym for the 20th cycle.

§ 4. Gleissberg's Forecast

In order to forecast the main characteristics of the 20th solar cycle, Gleissberg made use of a modification of the probability method developed by him previously. This modification consists in introducing basic relations from which the main characteristics of the next 11-year cycle may be determined with maximum possible probabilities of 90 and 95% (Gleissberg, 1952).

As previously, the method is based on a determination of the trend of the variation in the parameter $t_r^{(4)}$. If this quantity increases, then the upper limits of the other characteristics of the 11-year cycle, such as the height, are determined, and vice versa.

Let the number of the forthcoming solar cycle be n, so that the number of the current cycle is $n-1$ and those of the three preceding cycles are $n-2$, $n-3$, $n-4$. The quantity $t_r^{(4)}$ is here the average reduced length of the rising parts of cycles $n-4$, $n-3$, $n-2$, and -1. As a first approximation, let us assume that $t_r^{(4)}$ remains the same for cycles $n-3$, $n-2$, $n-1$, and n as well, and let us denote it in this case as $t_r^{(4)*}$. Finally, $S_3(x)$ is defined as the sum of the values of some parameter x for the three cycles $n-3$, $n-2$, and $n-1$.

If the maximum of cycle $n-1$ has passed, then we know its W_m, W_M, t_r, and t_l. On the basis of these data, we can give a forecast for the next cycle, with probabilities of 90 and 95%, using the following formulas:

with a probability of 90%

$$\left.\begin{aligned} t_r &\lesssim 4t_r^{(4)*} - S_3(t_r) \mp 2, \\ W_M &\,{}^{>1100}_{<1120} - 20t_r^{(4)*} - S_3(W_M), \\ t_l &\gtrsim 10t_r^{(4)*} - S_3(t_l)\,{}^{-160}_{-170}, \\ W_m &\,{}^{>114.0}_{<122.0} - 2.82t_r^{(4)*} - S_3(W_m), \end{aligned}\right\} \tag{5.1}$$

with a probability of 95%

$$\left.\begin{aligned} t_r &\lesssim 4t_r^{(4)*} - S_3(t_r) \mp 1, \\ W_M &\,{}^{>1080}_{<1140} - 20t_r^{(4)*} - S_3(W_M), \\ t_l &\gtrsim 10t_r^{(4)*} - S_3(t_l)\,{}^{-150}_{-180}, \\ W_m &\,{}^{>110.5}_{<125.5} - 2.82t_r^{(4)*} - S_3(W_m). \end{aligned}\right\} \tag{5.2}$$

It should be noted that these probabilities of 90 and 95% refer not to the entire forecast for the next cycle but only to each of the given relations individually.

If for cycle $n-1$ we also know $\frac{1}{4}W_M$ along the descending part of the cycle, then we know the value of t_f, and so the forecast for the next cycle can be supplemented by the prediction of t_f, using one of the following relations:

with a probability of 90%

$$t_f\,{}^{>385}_{<390} - 5t_r^{(4)*} - S_3(t_f), \tag{5.3}$$

with a probability of 95%

$$t_f\,{}^{>380}_{<395} - 5t_r^{(4)*} - S_3(t_f). \tag{5.4}$$

Let us now proceed directly to Gleissberg's forecast for the next cycle (Gleissberg, 1960). As shown by Table 9, for cycles 16 through 19 we have $W_M^{(4)} = 137.6$, $t_r^{(4)} = 29$, and $t_l^{(4)} = 40$. Thus, the preceding value of $W_M^{(4)}$ gives for the 20th cycle $W_M < 78.8$ (in the case of the lowest cycle). We also see from Table 9 that $t_r^{(4)}$ can be expected to increase in the future. Then the height of the next (20th) sunspot cycle can be forecast using the formula

$$W_M < 1140 - 20t_r^{(4)*} - S_3(W_M).$$

Here we can make use of the following data:

$$t_r^{(4)*} = 29; \quad S_3(W_M) = 119.2 + 154.8 + 201.3 = 472.3.$$

Consequently, it follows that, with a probability of 95%, we may expect that for the 20th cycle

$$W_M < 87.7.$$

If $W_M^{(4)} = 137.6$ is increased very slightly, then we still obtain a satisfactory agreement with this forecast. As mentioned in the previous section, the average maximum Wolf number for the last twenty solar cycles is 108. Therefore, according to Gleissberg, the height of the forthcoming 20th cycle will be lower than the average.

In the same article Gleissberg gives a more accurate value of t_l for the 19th sunspot cycle. He claims that, with a probability of 95%, the smoothed monthly Wolf number will reach $^1/_4\ W_M$ no later than May 1963.

We may use formulas (5.2) to calculate the other characteristics (t_r and W_m) for the 20th cycle and also t_l for the 19th cycle. Since $S_3(t_r) = 23+21+ +31 = 75$, we have for the 20th cycle $t_r < 42$. In order to determine the epoch of maximum of the 20th cycle, we need to know t_l for the 19th cycle. From Table 9 we have $S_3(t_l) = 45+37+41 = 123$, that is $t_l = 17$, so that in the next cycle the smoothed monthly Wolf number will reach $^1/_4 W_M(20) \approx 22$ no earlier than January 1965. By means of formula (4.6), we may now calculate the value of a, which determines the epoch of minimum of the next cycle. If we take $t_l > 17$, then we find that $a > 8$, so that the epoch of minimum of the 20th cycle will occur no later than January 1964.

According to the Zurich data, $S_3(W_m) = 3.4+7.7+3.4 = 14.5$. Consequently, using the last formula in (5.2), we find that the minimum Wolf number in the 20th sunspot cycle will be less than 29.2, a value which is in our opinion highly improbable. This improbability is apparently the reason why Gleissberg did not quote it in his forecast.

It is also possible that the other characteristics calculated here for the 20th sunspot cycle using his method led to doubts in Gleissberg's mind. For example, if we calculate $t_l + t_r = 42+17 = 59$, then we find that the epoch of maximum of the 20th cycle occurs in April 1968. This result coincides with the epoch of maximum given without any evidence by Minnis. However, even the formulas of Waldmeier (1955) set the epoch of maximum in 1969, with a much higher height assumed for the 20th cycle (Vitinskii, 1961a).

The height of the 20th cycle predicted by Gleissberg is chiefly determined by the heights of the twenty preceding 11-year cycles, that is, it is assumed implicitly that the next cycle will necessarily resemble one of these twenty cycles. In this sense, Gleissberg contradicts to some extent his own conclusion concerning the existence of a long-duration solar cycle (Gleissberg, 1944b). Finally, the preceding calculations show clearly how illusory are the probabilities used by Gleissberg for the predicted characteristics of the 11-year sunspot cycle.

§ 5. The Forecast of Ol'

Contrary to the preceding forecasts for the 20th sunspot cycle, the forecast of Ol' estimates not only the height but also the epochs of maximum and minimum. Since the prediction of the epochs of extrema is the newest and most interesting facet of this forecast, let us first consider the method used by Ol' to determine these epochs for the 11-year solar cycles.

Newcomb (1901) showed that the most probable hypothesis, in this respect, is the one which maintains that the temporal development of solar activity is governed by a strict periodicity, with a period of 11.13 years.

In order to verify Newcomb's hypothesis, Ol' (1960) calculated the epochs of extrema for the Wolf numbers from 1700 to 1901, using this period, and compared the values obtained with the observed epochs, determined using smoothed monthly Wolf numbers with an accuracy up to 0.1 year. The differences $\overline{\Delta t_{max}}$ and $\overline{\Delta t_{min}}$ obtained in this way were then averaged over four cycles. Table 19 gives the averaged values Δt_{max} and Δt_{min}, and also the values of $\overline{W_M}$ calculated using the same method.

TABLE 19

Deviations from epochs of extrema of the 11-year cycle (according to Ol')

Cycle numbers	$\overline{\Delta t_{min}}$	$\overline{\Delta t_{max}}$	$\overline{W_M}$
−3, − 2, −1, 0	+ 0.95	+ 1.45	78
1, 2, 3, 4	−1.72	−2.30	120
5, 6, 7, 8	−0.20	+ 0.90	76
9, 10, 11, 12	−0.02	−0.10	106
13, 14, 15, 16	+ 1.60	+ 1.20	83
17, 18, 19	−0.66	−1.40	152

An examination of Table 19 leads to the following conclusions:

1. there is a regular alternation of positive and negative values of $\overline{\Delta t_{max}}$ in successive cycle groups;
2. there is a positive correlation between the values of $\overline{\Delta t_{max}}$ and $\overline{\Delta t_{min}}$ for the corresponding cycle groups;
3. there is a negative correlation between $\overline{\Delta t_{max}}$ and $\overline{\Delta t_{min}}$, on the one hand, and $\overline{W_M}$, on the other;
4. there is no systematic increase in the values of $\overline{\Delta t_{max}}$ and $\overline{\Delta t_{min}}$ for later cycles.

Therefore, Newcomb's hypothesis is justified, with the minor addition that the deviations of the observed epochs of extrema for the Wolf numbers from the epochs calculated on the basis of the 11.13-year period are not random, but rather that they depend on the overall level of solar activity. This factor enabled Ol' to derive some correlation relations which could be used subsequently to forecast the epochs of minimum and maximum for the next 11-year sunspot cycle.

Ol' obtained the following coefficients r for the correlation between Δt_{max} for a given even cycle and Δt_{min} for the following cycles:

1) for the preceding odd cycle, + 0.60,
2) for the current even cycle, + 0.77,
3) for the next odd cycle, + 0.77.

Correspondingly, it was found that the coefficients for correlation between Δt_{max} for a given odd cycle, on the one hand, and Δt_{min} for the same odd cycle and for the next even cycle, on the other, are + 0.72 and + 0.75, respectively.

The coefficients for correlation between Δt_{max} and the values of W_M and $\sum W$ are also quite high: they are respectively -0.78 ± 0.08 and -0.72 ± 0.10. It was also found that W_M for a given cycle and Δt_{min} for the next cycle can be correlated stochastically, with a correlation coefficient of -0.64 ± 0.12, whereas there is virtually no correlation between Δt_{min} and W_M for any given cycle.

A very close correlation was established between Δt_{max} and $\Sigma_2 W$ (with $r = -0.91$). For the totality of even cycles this correlation is somewhat higher ($r = 0.93$) than for the odd cycles ($r = -0.78$). On the basis of all these correlations, Ol' obtained the following formulas for forecasting the epochs of extrema of the next (20th) solar cycle:

$$(\Delta t_{min})^{even}_{next} = 0.625\,(\Delta t_{max})^{odd.} - 0.25, \tag{5.5}$$

$$(\Delta t_{max})^{even} = 0.925\,(\Delta t_{min})^{odd}_{prec.} - 0.13. \tag{5.6}$$

For the current (19th) cycle, $\Delta t_{max} = -2.5$ and $\Delta t_{min} = -1.5$. Thus, using formulas (5.5) and (5.6), we find that $(\Delta t_{min})_{20} = 1.8$ and $(\Delta t_{max})_{20} = -1.5$. Let us next extend Newcomb's series of epochs of extrema for the Wolf numbers, constructed on the basis of the 11.13-year period, up to the beginning of the next century (see Table 20).

TABLE 20

Cycle number	Epoch of minimum	Epoch of maximum
19	1955.8	1960.4
20	1967.0	1971.5
21	1978.1	1982.6
22	1989.2	1993.8
23	2000.3	2004.9

Now, taking into account the values of $(\Delta t_{min})_{20}$ and $(\Delta t_{max})_{20}$, we find that the epoch of minimum of the next (20th) sunspot cycle should occur in $1967.0 - 1.8 = 1965.2$, while the epoch of maximum should be in $1971.5 - 1.5 = 1970.0$.

In accordance with the general tendency toward decreasing solar activity, we may expect values of Δt_{min} and Δt_{max} for the 21st and 22nd cycles which are close to zero, that is, the epochs of extrema of these cycles will be close to those given in Table 18 [apparently, Table 20 is meant here]. In the 23rd cycle these quantities may already become positive.

In order to estimate the height of the 20th cycle, Ol' (1961) used, in general, the method discussed in Chapter IV, § 4. The extrapolated value of $W_M^{(4)}$ for the 20th cycle can be taken as 125, and since we know that ΣW_M for cycles 17, 18, and 19 is 456 we obtain

$$W_M(20) = 125 \cdot 4 - 456 = 44.$$

Thus we have some indication that the 20th solar cycle will be analogous to the very low 5th cycle, which followed a cycle having the very high value of $\sum W = 840$. Actually, from the estimate of Ol' for the descending part of the cycle, we have $\sum W(19) \approx 1000$, that is, in this respect the 19th cycle is similar to the 4th cycle.

Independent (although very rough) estimates of W_M for the 20th sunspot cycle can be obtained using regressions between Δt_{max} and Δt_{min} and W_M. It was found that between $\delta t = \Delta t_{max} - \Delta t_{min}$ and W_M (for even cycles) the following regression formula is valid:

$$W_M = -31.4\delta t + 83 \quad (r = -0.61). \tag{5.7}$$

If we take the sums $\sum W_M$ and $\sum \delta t$ for neighboring cycles, combined into odd-even pairs, then we obtain the relation

$$\sum W_M = -21.5 \sum \delta t + 196. \qquad (5.8)$$

From the previously obtained values $(\Delta t_{min})_{20} = -1.8$ and $(\Delta t_{max})_{20} = -1.5$, we find that $\delta t = 0.3$. Then, after substituting this value into the last two regression formulas, taking into account that for the 19th cycle $W_M = 190$ and $\delta t = -2.6$, we find from the first regression that $W_M(20) = 74$ and from the second that $W_M(20) = 23$. The average of these is $W_M(20) = 48$, a value which is in satisfactory agreement with the estimate obtained using the method for ultralong-range forecasting developed previously by Ol'.

The forecast of Ol' makes use of certain unique procedures, and an attempt is made to achieve an intercontrol between the estimates obtained. Of particular interest is the method for the independent determination of the epochs of extrema of the 11-year cycles. Since Waldmeier's regression correlating the height of the cycle maximum with the average lengths of its rising and descending parts has been found to give a much lower accuracy than when initially applied, therefore this new method is especially valuable.

However, the forecast for the height of the next (20th) cycle, especially using equations (5.7) and (5.8), appears to be rather poor. According to this forecast, the maximum Wolf number will drop by about 140 to 150 from the present cycle to the next cycle. No comparable drop has been observed during the entire period of regular telescopic observations of sunspots (since 1700). On the other hand, Ol' uses the maximum relative spot numbers for this period only, and therefore he cannot obtain any other result. If he had used ΔW_M only, then analogously he could not have obtained a difference greater than 92. Thus, it may be concluded that cycles of higher order must be taken into consideration. Unfortunately, the lack of available data at present prohibits any progress in this direction, except for purely qualitative estimates.

Finally, it should be noted in passing that the height of the 20th cycle obtained by Ol' using equation (5.7) is in good agreement with Gleissberg's estimates, provided we keep in mind that Gleissberg gives smoothed monthly Wolf numbers rather than the yearly numbers.

§ 6. Vitinskii's Forecast

The forecast of Vitinskii (1961a) is qualitative in nature and is based on the data of Schove's table (see Table 12). According to this table, an 11-year cycle with an intensity comparable to that of the current cycle last occurred in 1369—1378 (about 600 years ago). On the other hand, it is known that in the 17th century solar activity was at an all-time low for the entire period of telescopic solar observations. In Chapter I, § 11 we gave some arguments in favor of the existence of a 600-year cycle, so that there is no point in repeating them here.

As observed previously (Chapter 4, § 7), Schove's data give $W_M = 85$ to 120 and $W_M = 145$, respectively, for the 20th and 21st cycles, if the arbitrary notation used by Schove is replaced by the average heights of the 11-year-cycle maxima. It seems likely that, since the current cycle is comparable in intensity to the cycle of 1369—1378, the height of the 20th sunspot cycle can be expected to be 100, as follows from Schove's table for the 14th century. Let us note, too, that this value falls within the range of W_M values predicted by Schove for the 20th cycle.

In order to forecast the height of the 21st solar cycle, we may use regression equation (1.19) (see Chapter I, § 7). From this equation, taking the height of the 20th cycle as 100, we find a value of W_M= 126 for the 21st cycle. In this forecast the epochs of extrema obtained for the 20th sunspot cycle by Ol' are used. The method of Ol' has unfortunately not yet been applied to the 21st cycle. Therefore, with great reservation of course, we apply the corresponding equations of Waldmeier. Using these equations, we obtain for the 21st solar cycle an epoch of minimum in 1976.0 and an epoch of maximum in 1979.4.

Previously, Vitinskii tried to forecast the height of the 20th cycle from the variations over many years of coefficients a and F and the oscillation of coefficient b in the Stewart-Panofsky formula (see Chapter I, § 8). Without mentioning that this method does not take into account long-duration solar cycles either, let us note that it leads to very contradictory results. The height of the 20th cycle is found to range from 50 to 120. This is due to the fact that the oscillations of coefficient b are extremely uncertain, whereas any slight change in the value of this coefficient will have an appreciable effect on the result. Therefore, although this method of forecasting is of some interest, the data at hand still are not sufficient to permit a successful application, and so this forecast can not be taken seriously.

§ 7. Baur's Forecast

Baur (1961) predicts the value of one characteristic of the 20th cycle, namely its epoch of minimum. Baur's method is based on the quite close correlation between the ratio $\frac{\tau}{t}$ of the lengths of the rising and descending parts of the cycle and the maximum smoothed monthly Wolf number W_M, obtained from the data for eighteen 11-year sunspot cycles.

Let us define $X_1=\frac{\tau}{t}$, $X_2=W_M$, where x_1 and x_2 are the deviations from the average values $\bar{X}_1$ and $\bar{X}_2$. The average length of an 11-year cycle is taken as 11.06 years. A study of the correlation between X_1 and X_2 for odd cycles has shown that in this case it is linear and quite high (r= +0.88). In spite of the small volume of data (n=9), this correlation coefficient is real, since P= 0.0027. Given r_{12} and σ_1 = 0.61 and σ_2= 32.14, let us find the mathematical expectation for the deviations x_1 from $\bar{X}_1$ in odd cycles:

$$E(x_1)=0.0167. \qquad (5.9)$$

In the current (19th) cycle, W_M= 200.8 and x_2 = + 95.2, so that $E(x_1)$= +1.59. Since $\bar{X}_1$ for the 9 odd cycles is 1.50, we obtain

$$E\left(\frac{\tau}{t}\right)=3.09. \qquad (5.10)$$

For the 19th cycle, t = 1957.9—1954.3 = 3.6, so that from (5.10) τ is found to be 11.1 years, that is, the next sunspot minimum will occur in 1969.0

and the duration of the present solar cycle will be 14.7 years. A comparable length has not been observed during the last 210 years.

Baur maintains that this result is due to the fact that for the highest odd 11-year cycles the relation between x_1 and x_2 is actually nonlinear. In order to overcome this difficulty, he introduces a new characteristic X_3, which is defined as follows. If t' and τ' are, respectively, the reduced lengths of the rising and descending parts of the 11-year cycle, from the epoch of maximum to the last month in which the smoothed Wolf number is less than or greater than $^1/_2\, W_M$, then $X_3 = \frac{\tau'}{t'}$. For odd cycles, the coefficient for the correlation between X_1 and X_3 is $r_{13} = +0.80$.

An additional parameter is then introduced to obtain a more real coefficient for the correlation with X_1, namely

$$X_4 = \sqrt{X_2 \cdot X_3}.$$

The coefficient for the correlation between X_1 and X_4 is found to be $r_{14} = +0.86$, so that in this case it can be considered real for $n = 9$. The average value $\bar{X}_4$ over the 9 odd 11-year cycles is found to be 12.92, with $\sigma_4 = 5.78$. According to the values of r_{14}, σ_1, and σ_4, we obtain

$$E(x_1) = 0.091 x_4, \tag{5.11}$$

where x_4 is the deviation of X_4 from the average value $\bar{X}_4$.

For the current (19th) cycle $t' = 1.75$. In order to estimate τ', Baur used the smoothed monthly relative spot numbers up to May 1960, and after extrapolation he obtained $\tau' = 2.92$. Thus, $X_3 = 1.67$ and $X_4 = 18.31$, so that $x_4 = 5.39$. When this value of x_4 is substituted into (5.11), we obtain $E(x_1) = +0.49$. Then, $X_1 = 1.99$, and since in the current cycle $t = 3.6$ we find that $\tau = 7.2$ years, that is, the epoch of minimum of the 20th sunspot cycle should be expected in February 1965. Since the expected error in X_2 is 0.21, therefore when the error is taken into account we see that the epoch of minimum will occur between April 1964 and October 1965.

It was initially assumed that linear correlations between X_1 and X_2 and between X_1 and X_3 exist for odd 11-year cycles only. Let us now forget this restriction and use 18 cycles rather than 9 cycles. For the 18 cycles we have

$$\bar{X}_1 = 1.61, \quad \bar{X}_4 = 12.88, \quad \sigma_1 = 0.59, \; \sigma_4 = 0.59, \quad r_{14} = +0.79.$$

The coefficient r_{14} is somewhat lower for all 18 cycles than it was for the 9 odd cycles. However, its reality is much more likely than for the case of odd cycles only.

Similarly to the foregoing, we find that

$$E(x_1) = 0.090 x_4. \tag{5.12}$$

Since in the 19th sunspot cycle $X_4 = 18.31$, therefore we have

$$E\left(\frac{\tau}{t}\right) = 2.10. \tag{5.13}$$

The expected error is 0.24. Thus we find that the epoch of minimum of the 20th cycle is in June/July 1965 and that the margin of error for this epoch extends it from August 1964 to April 1966.

Since it is still not clear which of the two alternatives of this method is better, Baur resorts to the following artificial device. He selects the time interval in which the error is common for the two alternatives (August 1964 through October 1965) and continues it symmetrically in both directions, until it corresponds to the margin of error for the first case. Then, he finds that the next sunspot minimum should be expected between June 1964 and December 1965. This result can also be obtained if, after calculating the epochs of minimum using $E(x_1)$ for the two alternatives, we then take their arithmetic mean, which is 1965.3 (April 1965), and finally measure off on either side of this epoch the margin of error corresponding to the first case.

Baur's forecast is quite close to that of Ol', although the former is based on the "eruption" hypothesis, while the latter is based on the properties of the 80-year to 90-year solar cycle. It should be noted, however, that Baur was unable to use as large a volume of data as Ol', because of the special features of his method. Baur's results should thus be regarded as less reliable. Nevertheless, we have given them here not only because there are still too few forecasts for the 20th cycle to allow us to discriminate but also as an independent confirmation of the results obtained previously by Ol'.

In conclusion, it should be pointed out, too, that Baur's method resorts to mathematical methods involving certain very artificial features, and this is not justified by the accuracy of the results obtained. Therefore, the method cannot be recommended, in our opinion, for actual application.

§ 8. A Summary of the Results

This last section represents a summary of all available forecasts for the main characteristics of the 20th sunspot cycle (Table 21).

TABLE 21

Author	W_M	Epoch of minimum	Epoch of maximum
Schove . . .	85–120	1966.5	1972.5
Bezrukova . .	<65–75		
Minnis . . .	110–160		
Gleissberg . .	<88		1968
Ol'	44–48	1965.2	1970.0
Vitinskii. . .	100		
Baur		1964.5–1965.9	

It is still too soon to attempt any kind of analysis of this table, and only the future can tell to what extent the forecasts discussed here will be successful. In the meantime, as long as the forecasts are so few, readers are referred to the relevant articles which will undoubtedly appear in the near future.

CONCLUSION

We have considered the principal empiricostatistical methods for the long-range forecasting of Wolf numbers and we have shown that the reliability of the results obtained using these methods still leaves much to be desired. What, then, are the possible ways in which solar-activity forecasts can be developed in the future?

First, a comprehensive approach to this problem must be worked out. Even the most perfect theory of solar activity, if it were developed in the very near future, would not give completely reliable results. However, at present we still do not have anything which remotely resembles such a theory, so that as a first stage we should seek some way in which satisfactory results can be obtained even using just the individual theoretical conclusions of solar physics.

The methods which were discussed here are virtually independent of the morphology of solar activity. Nevertheless, a morphological approach to solar-activity forecasting would in our opinion mean a real advance. This is particularly important with respect to the development of short-range forecasting. A morphological approach involves a study of the development of centers of activity, that is, of all the layers of the solar atmosphere from bottom to top. The future course of sunspot activity can then be evaluated according to certain preliminary variations which take place in the other layers of the solar atmosphere. Moreover, observations of solar radio emission in the centimeter range give us a look into the invisible hemisphere of the sun a day or two before a spot group emerges over the eastern limb onto the visible hemisphere (Molchanov, 1959; Ikhsanova, 1960). Consequently, these data provide us with a means for the short-range forecasting of solar activity.

The morphological approach has still not been tested much with respect to monthly forecasts. If it has been used at all, it has been regarded just as being supplementary to statistical methods. Thus, it would be very interesting to develop a morphological method for monthly forecasting which could have equal status with the existing statistical method of Mayot.

At present, the only subject of solar-activity forecasting (not counting the spot areas) is the Wolf numbers. However, except in ultralong-range forecasts, it would be very useful to develop methods for forecasting various different indexes of solar activity. Then, parallel forecasts of different indexes may even be used for an internal control of the reliability, if not for every parameter, then at any rate for many of them.

Finally, it is of exceptional importance for forecasting that a comprehensive theory of solar activity be developed. Such a theory can be used as a basis for the development of theoretical methods for forecasting the solar indexes. Although the first attempt to do this, that of Rubashev (1954), failed to give satisfactory results, still the fact that this problem was posed is a very valuable thing. It is hoped that in the near future science will be able to solve the main enigma related to the sun, namely to determine the reason for solar activity and to ascertain its mechanism. Quite recently, Bjerknes's hydrodynamic theory of solar activity (Bjerknes, 1926) still seemed plausible. Then, however, Alfvén (1952) published a theory which completely refuted Bjerknes's picture of solar activity and substituted magnetohydrodynamic waves for it. At present a synthesis of these two trends is in process, and this has led to a vigorous development of magnetohydrodynamics. This is not the place to discuss the many problems which are being considered or have already been solved in this new field of physics and astrophysics. However, it should be mentioned that now hardly anyone doubts the basic fact that without taking magnetic phenomena into account it is impossible to construct a valid theory of solar activity. On the other hand, various investigators now pay special attention to the study of the properties of differential solar rotation and of its relation to the magnetic energy of the sun.

These are, in brief, the main trends observed in the solution of the problem of solar-activity forecasting. In order to summarize using even fewer words, let us just mention that all studies of active processes taking place on the sun, whatever the subject of these studies may be, will contribute toward a solution of this important problem.

TABLE 1

Monthly and yearly observations of Wolf numbers

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1749	58.0	62.6	70.0	55.7	85.0	83.5	94.8	66.3	75.9	75.5	158.6	85.2	80.9
1750	73.3	75.9	89.2	88.3	90.0	100.0	85.4	103.0	91.2	65.7	63.3	75.4	83.4
1751	70.0	43.5	45.3	56.4	60.7	50.7	66.3	59.8	23.5	23.2	28.5	44.0	47.7
1752	35.0	50.0	71.0	59.3	59.7	39.6	78.4	29.3	27.1	46.6	37.6	40.0	47.8
1753	44.0	32.0	45.7	38.0	36.0	31.7	22.0	39.0	28.0	25.0	20.0	6.7	30.7
1754	0.0	3.0	1.7	13.7	20.7	26.7	18.8	12.3	8.2	24.1	13.2	4.2	12.2
1755	10.2	11.2	6.8	6.5	0.0	0.0	8.6	3.2	17.8	23.7	6.8	20.0	9.6
1756	12.5	7.1	5.4	9.4	12.5	12.9	3.6	6.4	11.8	14.3	17.0	9.4	10.2
1757	14.1	21.2	26.2	30.0	38.1	12.8	25.0	51.3	39.7	32.5	64.7	33.5	32.4
1758	37.6	52.0	49.0	72.3	46.4	45.0	44.0	38.7	62.5	37.7	43.0	43.0	47.6
1759	48.3	44.0	46.8	47.0	49.0	50.0	51.0	71.3	77.2	59.7	46.3	57.0	54.0
1760	67.3	59.5	74.7	58.3	72.0	48.3	66.0	75.6	61.3	50.6	59.7	61.0	62.9
1761	70.0	91.0	80.7	71.7	107.2	99.3	94.1	91.1	100.7	88.7	89.7	46.0	85.9
1762	43.8	72.8	45.7	60.2	39.9	77.1	33.8	67.7	68.5	69.3	77.8	77.2	61.2
1763	56.5	31.9	34.2	32.9	32.7	35.8	54.2	26.5	68.1	46.3	60.9	61.4	45.1
1764	59.7	59.7	40.2	34.4	44.3	30.0	30.0	30.0	28.2	28.0	26.0	25.7	36.4
1765	24.0	26.0	25.0	22.0	20.2	20.0	27.0	29.7	16.0	14.0	14.0	13.0	20.9
1766	12.0	11.0	36.6	6.0	26.8	3.0	3.3	4.0	4.3	5.0	5.7	19.2	11.4
1767	27.4	30.0	43.0	32.9	29.8	33.3	21.9	40.8	42.7	44.1	54.7	53.3	37.8
1768	53.5	66.1	46.3	42.7	77.7	77.4	52.6	66.8	74.8	77.8	90.6	111.8	69.8
1769	73.9	64.2	64.3	96.7	73.6	94.4	118.6	120.3	148.8	158.2	148.1	112.0	106.1
1770	104.0	142.5	80.1	51.0	70.1	83.3	109.8	126.3	104.4	103.6	132.2	102.3	100.8
1771	36.0	46.2	46.7	64.9	152.7	119.5	67.7	58.5	101.4	90.0	99.7	95.7	81.6
1772	100.9	90.8	31.1	92.2	38.0	57.0	77.3	56.2	50.5	78.6	61.3	64.0	66.5
1773	54.6	29.0	51.2	32.9	41.1	28.4	27.7	12.7	29.3	26.3	40.9	43.2	34.8
1774	46.8	65.4	55.7	43.8	51.3	28.5	17.5	6.6	7.9	14.0	17.7	12.2	30.6
1775	4.4	0.0	11.6	11.2	3.9	12.3	1.0	7.9	3.2	5.6	15.1	7.9	7.0

TABLE 1 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1776	21.7	11.6	6.3	21.8	11.2	19.0	1.0	24.2	16.0	30.0	35.0	40.0	19.8
1777	45.0	36.5	39.0	95.5	80.3	80.7	95.0	112.0	116.2	106.5	146.0	157.3	92.5
1778	177.3	109.3	134.0	145.0	238.9	171.6	153.0	140.0	171.7	156.3	150.3	105.0	154.4
1779	114.7	165.7	118.0	145.0	140.0	113.7	143.0	112.0	111.0	124.0	114.0	110.0	125.9
1780	70.0	98.0	98.0	95.0	107.2	88.0	86.0	86.0	93.7	77.0	60.0	58.7	84.8
1781	98.7	74.7	53.0	68.3	104.7	97.7	73.5	66.0	51.0	27.3	67.0	35.2	68.1
1782	54.0	37.5	37.0	41.0	54.3	38.0	37.0	44.0	34.0	23.2	31.5	30.0	38.5
1783	28.0	38.7	26.7	28.3	23.0	25.2	32.2	20.0	18.0	8.0	15.0	10.5	22.8
1784	13.0	8.0	11.0	10.0	6.0	9.0	6.0	10.0	10.0	8.0	17.0	14.0	10.2
1785	6.5	8.0	9.0	15.7	20.7	26.3	36.3	20.0	32.0	47.2	40.2	27.3	24.1
1786	37.2	47.6	47.7	85.4	92.3	59.0	83.0	89.7	111.5	112.3	116.0	112.7	82.9
1787	134.7	106.0	87.4	127.2	134.8	99.2	128.0	137.2	157.3	157.0	141.5	174.0	132.0
1788	138.0	129.2	143.3	108.5	113.0	154.2	141.5	136.0	141.0	142.0	94.7	129.5	130.9
1789	114.0	125.3	120.0	123.3	123.5	120.0	117.0	103.0	112.0	89.7	134.0	135.5	118.1
1790	103.0	127.5	96.3	94.0	93.0	91.0	69.3	87.0	77.3	84.3	82.0	74.0	89.9
1791	72.7	62.0	74.0	77.2	73.7	64.2	71.0	43.0	66.5	61.7	67.0	66.0	66.6
1792	58.0	64.0	63.0	75.7	62.0	61.0	45.8	60.0	59.0	59.0	57.0	56.0	60.0
1793	56.0	55.0	55.5	53.0	52.3	51.0	50.0	29.3	24.0	47.0	44.0	45.7	46.9
1794	45.0	44.0	38.0	28.4	55.7	41.5	41.0	40.0	11.1	28.5	67.4	51.4	41.0
1795	21.4	39.9	12.6	18.6	31.0	17.1	12.9	25.7	13.5	19.5	25.0	18.0	21.3
1796	22.0	23.8	15.7	31.7	21.0	6.7	26.9	1.5	18.4	11.0	8.4	5.1	16.0
1797	14.4	4.2	4.0	4.0	7.3	11.1	4.3	6.0	5.7	6.9	5.8	3.0	6.4
1798	2.0	4.0	12.4	1.1	0.0	0.0	0.0	3.0	2.4	1.5	12.5	9.9	4.1
1799	1.6	12.6	21.7	8.4	8.2	10.6	2.1	0.0	0.0	4.6	2.7	8.6	6.8
1800	6.9	9.3	13.9	0.0	5.0	23.7	21.0	19.5	11.5	12.3	10.5	40.1	14.5
1801	27.0	29.0	30.0	31.0	32.0	31.2	35.0	38.7	33.5	32.6	39.8	48.2	34.0
1802	47.8	47.0	40.8	42.0	44.0	46.0	48.0	50.0	51.8	38.5	34.5	50.0	45.0
1803	50.0	50.8	29.5	25.0	44.3	36.0	48.3	34.1	45.3	54.3	51.0	48.0	43.1
1804	45.3	48.3	48.0	50.6	33.4	34.8	29.8	43.1	53.0	62.3	61.0	60.0	47.5
1805	61.0	44.1	51.4	37.5	39.0	40.5	37.6	42.7	44.4	29.4	41.0	38.3	42.2
1806	39.0	29.6	32.7	27.7	26.4	25.6	30.0	26.3	24.0	27.0	25.0	24.0	28.1

TABLE 1 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1807	12.0	12.2	9.6	23.8	10.0	12.0	12.7	12.0	5.7	8.0	2.6	0.0	10.1
1808	0.0	4.5	0.0	12.3	13.5	13.5	6.7	8.0	11.7	4.7	10.5	12.3	8.1
1809	7.2	9.2	0.9	2.5	2.0	7.7	0.3	0.2	0.4	0.0	0.0	0.0	2.5
1810	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1811	0.0	0.0	0.0	0.0	0.0	0.0	6.6	0.0	2.4	6.1	0.8	1.1	1.4
1812	11.3	1.9	0.7	0.0	1.0	1.3	0.5	15.6	5.2	3.9	7.9	10.1	5.0
1813	0.0	10.3	1.9	16.6	5.5	11.2	18.3	8.4	15.3	27.8	16.7	14.3	12.2
1814	22.2	12.0	5.7	23.8	5.8	14.9	18.5	2.3	8.1	19.3	14.5	20.1	13.9
1815	19.2	32.2	26.2	31.6	9.8	55.9	35.5	47.2	31.5	33.5	37.2	65.0	35.4
1816	26.3	68.8	73.7	58.8	44.3	43.6	38.8	23.2	47.8	56.4	38.1	29.9	45.8
1817	36.4	57.9	96.2	26.4	21.2	40.0	50.0	45.0	36.7	25.6	28.9	28.4	41.1
1818	34.9	22.4	29.7	34.5	53.1	36.4	28.0	31.5	26.1	31.7	10.9	25.8	30.4
1819	32.5	20.7	3.7	20.2	19.6	35.0	31.4	26.1	14.9	27.5	25.1	30.6	23.9
1820	19.2	26.6	4.5	19.4	29.3	10.8	20.6	25.9	5.2	9.0	7.9	9.7	15.7
1821	21.5	4.3	5.7	9.2	1.7	1.8	2.5	4.8	4.4	18.8	4.4	0.0	6.6
1822	0.0	0.9	16.1	13.5	1.5	5.6	7.9	2.1	0.0	0.4	0.0	0.0	4.0
1823	0.0	0.0	0.6	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	20.4	1.8
1824	21.6	10.8	0.0	19.4	2.8	0.0	0.0	1.4	20.5	25.2	0.0	0.8	8.5
1825	5.0	15.5	22.4	3.8	15.4	15.4	30.9	25.4	15.7	15.6	11.7	22.0	16.6
1826	17.7	18.2	36.7	24.0	32.4	37.1	52.5	39.6	18.9	50.6	39.5	68.1	36.3
1827	34.6	47.4	57.8	46.0	56.3	56.7	42.9	53.7	49.6	57.2	48.2	46.1	49.7
1828	52.8	64.4	65.0	61.1	89.1	98.0	54.3	76.4	50.4	34.7	57.0	46.9	62.5
1829	43.0	49.4	72.3	95.0	67.5	73.9	90.8	78.3	52.8	57.2	67.6	56.5	67.0
1830	52.2	72.1	84.6	107.1	66.3	65.1	43.9	50.7	62.1	84.4	81.2	82.1	71.0
1831	47.5	50.1	93.4	54.6	38.1	33.4	45.2	54.9	37.9	46.2	43.5	28.9	47.8
1832	30.9	55.5	55.1	26.9	41.3	26.7	13.9	8.9	8.2	21.1	14.3	27.5	27.5
1833	11.3	14.9	11.8	2.8	12.9	1.0	7.0	5.7	11.6	7.5	5.9	9.9	8.5
1834	4.9	18.1	3.9	1.4	8.8	7.8	8.7	4.0	11.5	24.8	30.5	34.5	13.2
1835	7.5	24.5	19.7	61.5	43.6	33.2	59.8	59.0	100.8	95.2	100.0	77.5	56.9
1836	88.6	107.6	98.1	142.9	111.4	124.7	116.7	107.8	95.1	137.4	120.9	206.2	121.5
1837	188.0	175.6	134.6	138.2	111.3	158.0	162.8	134.0	96.3	123.7	107.0	129.8	138.3

TABLE 1 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1838	144.9	84.8	140.8	126.6	137.6	94.5	108.2	78.8	73.6	90.8	77.4	79.8	103.2
1839	107.6	102.5	77.7	61.8	53.8	54.6	84.7	131.2	132.7	90.8	68.8	63.6	85.8
1840	81.2	87.7	55.5	65.9	69.2	48.5	60.7	57.8	74.0	49.8	54.3	53.7	63.2
1841	24.0	29.9	29.7	42.6	67.4	55.7	30.8	39.3	35.1	28.5	19.8	38.8	36.8
1842	20.4	22.1	21.7	26.9	24.9	20.5	12.6	26.5	18.5	38.1	40.5	17.6	24.2
1843	13.3	3.5	8.3	8.8	21.1	10.5	9.5	11.8	4.2	5.3	19.1	12.7	10.7
1844	9.4	14.7	13.6	20.8	12.0	3.7	21.2	23.9	6.9	21.5	10.7	21.6	15.0
1845	25.7	43.6	43.3	56.9	47.8	31.1	30.6	32.3	29.6	40.7	39.4	59.7	40.1
1846	38.7	51.0	63.9	69.2	59.9	65.1	46.5	54.8	107.1	55.9	60.4	65.5	61.5
1847	62.6	44.9	85.7	44.7	75.4	85.3	52.2	140.6	161.2	180.4	138.9	109.6	98.5
1848	159.1	111.8	108.9	107.1	102.2	123.8	139.2	132.5	100.3	132.4	114.6	159.9	124.3
1849	156.7	131.7	96.5	102.5	80.6	81.2	78.0	61.3	93.7	71.5	99.7	97.0	95.9
1850	78.0	89.4	82.6	44.1	61.6	70.0	39.1	61.6	86.2	71.0	54.8	60.0	66.5
1851	75.5	105.4	64.6	56.5	62.6	63.2	36.1	57.4	67.9	62.5	50.9	71.4	64.5
1852	68.4	67.5	61.2	65.4	54.9	46.9	42.0	39.7	37.5	67.3	54.3	45.4	54.2
1853	41.1	42.9	37.7	47.6	34.7	40.0	45.9	50.4	33.5	42.3	28.8	23.4	39.0
1854	15.4	20.0	20.7	26.4	24.0	21.1	18.7	15.8	22.4	12.7	28.2	21.4	20.6
1855	12.3	11.4	17.4	4.4	9.1	5.3	0.4	3.1	0.0	9.7	4.2	3.1	6.7
1856	0.5	4.9	0.4	6.5	0.0	5.0	4.6	5.9	4.4	4.5	7.7	7.2	4.3
1857	13.7	7.4	5.2	11.1	29.2	16.0	22.2	16.9	42.4	40.6	31.4	37.2	22.8
1858	39.0	34.9	57.5	38.3	41.4	44.5	56.7	55.3	80.1	91.2	51.9	66.9	54.8
1859	83.7	87.6	90.3	85.7	91.0	87.1	95.2	106.8	105.8	114.6	97.2	81.0	93.8
1860	81.5	88.0	98.9	71.4	107.1	108.6	116.7	100.3	92.2	90.1	97.9	95.6	95.7
1861	62.3	77.8	101.0	98.5	56.8	87.8	78.0	82.5	79.9	67.2	53.7	80.5	77.2
1862	63.1	64.5	43.6	53.7	64.4	84.0	73.4	62.5	66.6	42.0	50.6	40.9	59.1
1863	48.3	56.7	66.4	40.6	53.8	40.8	32.7	48.1	22.0	39.9	37.7	41.2	44.0
1864	57.7	47.1	66.3	35.8	40.6	57.8	54.7	54.8	28.5	33.9	57.6	28.6	47.0
1865	48.7	39.3	39.5	29.4	34.5	33.6	26.8	37.8	21.6	17.1	24.6	12.8	30.5
1866	31.6	38.4	24.6	17.6	12.9	16.5	9.3	12.7	7.3	14.1	9.0	1.5	16.3
1867	0.0	0.7	9.2	5.1	2.9	1.5	5.0	4.9	9.8	13.5	9.3	25.2	7.3
1868	15.6	15.8	26.5	36.6	26.7	31.1	28.6	34.4	43.8	61.7	59.1	67.6	37.3

TABLE 1 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1869	60.9	59.3	52.7	41.0	104.0	108.4	59.2	79.6	80.6	59.4	77.4	104.3	73.9
1870	77.3	114.9	159.4	160.0	176.0	135.6	132.4	153.8	136.0	146.4	147.5	130.0	139.1
1871	88.3	125.3	143.2	162.4	145.5	91.7	103.0	110.0	80.3	89.0	105.4	90.3	111.2
1872	79.5	120.1	88.4	102.1	107.6	109.9	105.5	92.9	114.6	103.5	112.0	83.9	101.7
1873	86.7	107.0	98.3	76.2	47.9	44.8	66.9	68.2	47.5	47.4	55.4	49.2	66.3
1874	60.8	64.2	46.4	32.0	44.6	38.2	67.8	61.3	28.0	34.3	28.9	29.3	44.7
1875	14.6	22.2	33.8	29.1	11.5	23.9	12.5	14.6	2.4	12.7	17.7	9.9	17.1
1876	14.3	15.0	31.2	2.3	5.1	1.6	15.2	8.8	9.9	14.3	9.9	8.2	11.3
1877	24.4	8.7	11.7	15.8	21.2	13.4	5.9	6.3	16.4	6.7	14.5	2.3	12.3
1878	3.3	6.0	7.8	0.1	5.8	6.4	0.1	0.0	5.3	1.1	4.1	0.5	3.4
1879	0.8	0.6	0.0	6.2	2.4	4.8	7.5	10.7	6.1	12.3	12.9	7.2	6.0
1880	24.0	27.5	19.5	19.3	23.5	34.1	21.9	48.1	66.0	43.0	30.7	29.6	32.3
1881	36.4	53.2	51.5	51.7	43.5	60.5	76.9	58.0	53.2	64.0	54.8	47.3	54.3
1882	45.0	69.3	67.5	95.8	64.1	45.2	45.4	40.4	57.7	59.2	84.4	41.8	59.7
1883	60.6	46.9	42.8	82.1	32.1	76.5	80.6	46.0	52.6	83.8	84.5	75.9	63.7
1884	91.5	86.9	86.8	76.1	66.5	51.2	53.1	55.8	61.9	47.8	36.6	47.2	63.5
1885	42.8	71.8	49.8	55.0	73.0	83.7	66.5	50.0	39.6	38.7	33.3	21.7	52.2
1886	29.9	25.9	57.3	43.7	30.7	27.1	30.3	16.9	21.4	8.6	0.3	12.4	25.4
1887	10.3	13.2	4.2	6.9	20.0	15.7	23.3	21.4	7.4	6.6	6.9	20.7	13.1
1888	12.7	7.1	7.8	5.1	7.0	7.1	3.1	2.8	8.8	2.1	10.7	6.7	6.8
1889	0.8	8.5	7.0	4.3	2.4	6.4	9.7	20.6	6.5	2.1	0.2	6.7	6.3
1890	5.3	0.6	5.1	1.6	4.8	1.3	11.6	8.5	17.2	11.2	9.6	7.8	7.1
1891	13.5	22.2	10.4	20.5	41.1	48.3	58.8	33.2	53.8	51.5	41.9	32.2	35.6
1892	69.1	75.6	49.9	69.6	79.6	76.3	76.8	101.4	62.8	70.5	65.4	78.6	73.0
1893	75.0	73.0	65.7	88.1	84.7	88.2	88.8	129.2	77.9	79.7	75.1	93.8	84.9
1894	83.2	84.6	52.3	81.6	101.2	98.9	106.0	70.3	65.9	75.5	56.6	60.0	78.0
1895	63.3	67.2	61.0	76.9	67.5	71.5	47.8	68.9	57.7	67.9	47.2	70.7	64.0
1896	29.0	57.4	52.0	43.8	27.7	49.0	45.0	27.2	61.3	28.4	38.0	42.6	41.8
1897	40.6	29.4	29.1	31.0	20.0	11.3	27.6	21.8	48.1	14.3	8.4	33.3	26.2
1898	30.2	36.4	38.3	14.5	25.8	22.3	9.0	31.4	34.8	34.4	30.9	12.6	26.7
1899	19.5	9.2	18.1	14.2	7.7	20.5	13.5	2.9	8.4	13.0	7.8	10.5	12.1

TABLE 1 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1900	9.4	13.6	8.6	16.0	15.2	12.1	8.3	4.3	8.3	12.9	4.5	0.3	9.5
1901	0.2	2.4	4.5	0.0	10.2	5.8	0.7	1.0	0.6	3.7	3.8	0.0	2.7
1902	5.2	0.0	12.4	0.0	2.8	1.4	0.9	2.3	7.6	16.3	10.3	1.1	5.0
1903	8.3	17.0	13.5	26.1	14.6	16.3	27.9	28.8	11.1	38.9	44.5	45.6	24.4
1904	31.6	24.5	37.2	43.0	39.5	41.9	50.6	58.2	30.1	54.2	38.0	54.6	42.0
1905	54.8	85.8	56.5	39.3	48.0	49.0	73.0	58.8	55.0	78.7	107.2	55.5	63.5
1906	45.5	31.3	64.5	55.3	57.7	63.2	103.3	47.7	56.1	17.8	38.9	64.7	53.8
1907	76.4	108.2	60.7	52.6	43.0	40.4	49.7	54.3	85.0	65.4	61.5	47.3	62.0
1908	39.2	33.9	28.7	57.6	40.8	48.1	39.5	90.5	86.9	32.3	45.5	39.5	48.5
1909	56.7	46.6	66.3	32.3	36.0	22.6	35.8	23.1	38.8	58.4	55.8	54.2	43.9
1910	26.4	31.5	21.4	8.4	22.2	12.3	14.1	11.5	26.2	38.3	4.9	5.8	18.6
1911	3.4	9.0	7.8	16.5	9.0	2.2	3.5	4.0	4.0	2.6	4.2	2.2	5.7
1912	0.3	0.0	4.9	4.5	4.4	4.1	3.0	0.3	9.5	4.6	1.1	6.4	3.6
1913	2.3	2.9	0.5	0.9	0.0	0.0	1.7	0.2	1.2	3.1	0.7	3.8	1.4
1914	2.8	2.6	3.1	17.3	5.2	11.4	5.4	7.7	12.7	8.2	16.4	22.3	9.6
1915	23.0	42.3	38.8	41.3	33.0	68.8	71.6	69.6	49.5	53.5	42.5	34.5	47.4
1916	45.3	55.4	67.0	71.8	74.5	67.7	53.5	35.2	45.1	50.7	65.6	53.0	57.1
1917	74.7	71.9	94.8	74.7	114.1	114.9	119.8	154.5	129.4	72.2	96.4	129.3	103.9
1918	96.0	65.3	72.2	80.5	76.7	59.4	107.6	101.7	79.9	85.0	83.4	59.2	80.6
1919	48.1	79.5	66.5	51.8	88.1	111.2	64.7	69.0	54.7	52.8	42.0	34.9	63.6
1920	51.1	53.9	70.2	14.8	33.3	38.7	27.5	19.2	36.3	49.6	27.2	29.9	37.6
1921	31.5	28.3	26.7	32.4	22.2	33.7	41.9	22.8	17.8	18.2	17.8	20.3	26.1
1922	11.8	26.4	54.7	11.0	8.0	5.8	10.9	6.5	4.7	6.2	7.4	17.5	14.2
1923	4.5	1.5	3.3	6.1	3.2	9.1	3.5	0.5	13.2	11.6	10.0	2.8	5.8
1924	0.5	5.1	1.8	11.3	20.8	24.0	28.1	19.3	25.1	25.6	22.5	16.5	16.7
1925	5.5	23.2	18.0	31.7	42.8	47.5	38.5	37.9	60.2	69.2	58.6	98.6	44.3
1926	71.8	70.0	62.5	38.5	64.3	73.5	52.3	61.6	60.8	71.5	60.5	79.4	63.9
1927	81.6	93.0	69.6	93.5	79.1	59.1	54.9	53.8	68.4	63.1	67.2	45.2	69.0
1928	83.5	73.5	85.4	80.6	76.9	91.4	98.0	83.8	89.7	61.4	50.3	59.0	77.8
1929	68.9	64.1	50.2	52.8	58.2	71.9	70.2	65.8	34.4	54.0	81.1	108.0	65.0
1930	65.3	49.2	35.0	38.2	36.8	28.8	21.9	24.9	32.1	34.4	35.6	25.8	35.7

TABLE 1 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1931	14.6	43.1	30.0	31.2	24.6	15.3	17.4	13.0	19.0	10.0	18.7	17.8	21.2
1932	12.1	10.6	11.2	11.2	17.9	22.2	9.6	6.8	4.0	8.9	8.2	11.0	11.1
1933	12.3	22.2	10.1	2.9	3.2	5.2	2.8	0.2	5.1	3.0	0.6	0.3	5.7
1934	3.4	7.8	4.3	11.3	19.7	6.7	9.3	8.3	4.0	5.7	8.7	15.4	8.7
1935	18.9	20.5	23.1	12.2	27.3	45.7	33.9	30.1	42.1	53.2	64.2	61.5	36.1
1936	62.8	74.3	77.1	74.9	54.6	70.0	52.3	87.0	76.0	89.0	115.4	123.4	79.7
1937	132.5	128.5	83.9	109.3	116.7	130.3	145.1	137.7	100.7	124.9	74.4	88.8	114.4
1938	98.4	119.2	86.5	101.0	127.4	97.5	165.3	115.7	89.6	99.1	122.2	92.7	109.6
1939	80.3	77.4	64.6	109.1	118.3	101.0	97.6	105.8	112.6	88.1	68.1	42.1	88.8
1940	50.5	59.4	83.3	60.7	54.4	83.9	67.5	105.5	66.5	55.0	58.4	68.3	67.8
1941	45.6	44.5	46.4	32.8	29.5	59.8	66.9	60.0	65.9	46.3	38.3	33.7	47.5
1942	35.6	52.8	54.2	60.7	25.0	11.4	17.7	20.2	17.2	19.2	30.7	22.5	30.6
1943	12.4	28.9	27.4	26.1	14.1	7.6	13.2	19.4	10.0	7.8	10.2	18.8	16.3
1944	3.7	0.5	11.0	0.3	2.5	5.0	5.0	16.7	14.3	16.9	10.8	28.4	9.6
1945	18.5	12.7	21.5	32.0	30.6	36.2	42.6	25.9	34.9	68.8	46.0	27.4	33.2
1946	47.6	86.2	76.6	75.7	84.9	73.5	116.2	107.2	94.4	102.3	123.8	121.7	92.6
1947	115.7	133.4	129.8	149.8	201.3	163.9	157.9	188.8	169.4	163.6	128.0	116.5	151.6
1948	108.5	86.1	94.8	189.7	174.0	167.8	142.2	157.9	143.3	136.3	95.8	138.0	136.2
1949	119.1	182.3	157.5	147.0	106.2	121.7	125.8	123.8	145.3	131.6	143.5	117.6	135.1
1950	101.6	94.8	109.7	113.4	106.2	83.6	91.0	85.2	51.3	61.4	54.8	54.1	83.9
1951	59.9	59.9	55.9	92.9	108.5	100.6	61.5	61.0	83 1	51.6	52.4	45.8	69.4
1952	40.7	22.7	22.0	29.1	23.4	36.4	39.3	54.9	28.2	23.8	22.1	34.3	31.4
1953	26.5	3.9	10.0	27.8	12.5	21.8	8.6	23.5	19.3	8.2	1.6	2.5	13.9
1954	0.2	0.5	10.9	1.8	0.8	0.2	4.8	8.4	1.5	7.0	9.2	7.6	4.4
1955	23.1	20.8	4.9	11.3	28.9	31.7	26.7	40.7	42.7	58.5	89.2	76.9	38.0
1956	73.6	124.0	118.4	110.7	136.6	116.6	129.1	169.6	173.2	155.3	201.3	192.1	141.7
1957	165.0	130.2	157.4	175.2	164.6	200.7	187.2	158.0	235.8	253.8	210.9	239.4	189.9
1958	202.5	164.9	190.7	196.0	175.3	171.5	191.4	200.2	201.2	181.5	152.3	187.6	184.8
1959	217.4	143.1	185.7	163.3	172.0	168.7	149.6	199.6	145.2	111.4	124.0	125.0	159.0
1960	146.3	106.0	102.2	122.0	119.6	110.2	121.7	134.1	127.2	82.8	89.6	85.6	112.3
1961	57.9	46.1	53.0	61.4	51.0	77.4	70.2	55.8	63.6	37.7	32.6	39.9	53.9

TABLE 2

Monthly and yearly smoothed Wolf numbers

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1749							81.6	82.8	84.1	86.3	87.8	88.7	
1750	89.0	90.2	92.3	92.6	88.2	83.8	83.3	81.8	78.6	75.4	72.9	69.6	83.1
1751	66.8	64.2	59.5	54.9	51.7	49.0	46.2	45.0	46.4	47.5	47.6	47.1	52.2
1752	47.2	46.4	45.3	46.4	47.8	48.0	48.2	47.8	46.0	44.1	42.2	40.9	5.9
1753	38.2	36.2	36.7	35.8	34.2	32.1	28.8	25.8	22.8	19.9	18.3	17.4	28.9
1754	17.1	15.8	13.9	13.0	12.7	12.3	12.6	13.4	14.0	13.9	12.7	10.7	13.5
1755	9.2	8.4	8.4	8.8	8.5	8.9	9.7	9.6	9.4	9.4	10.1	11.1	9.3
1756	11.5	11.4	11.3	10.6	10.7	10.6	10.3	10.9	12.4	14.1	16.0	17.1	12.2
1757	18.0	20.7	23.8	25.7	28.4	31.4	33.4	35.7	37.9	40.6	42.7	44.4	31.9
1758	46.5	46.8	47.2	48.4	47.7	47.2	48.0	48.2	47.7	46.6	45.6	46.0	47.2
1759	46.5	48.1	50.1	51.5	52.7	53.4	54.8	56.2	58.0	59.6	61.1	62.0	54.5
1760	62.5	63.3	62.8	61.8	62.0	62.7	63.0	64.4	66.0	66.8	68.8	72.4	64.7
1761	75.7	77.5	79.8	83.0	85.9	86.5	84.8	82.9	80.7	78.8	75.5	71.7	80.2
1762	68.3	64.8	62.5	60.4	59.0	59.9	61.7	60.5	58.3	56.7	55.3	53.2	60.1
1763	52.4	51.5	49.8	48.8	47.1	45.8	45.3	46.5	48.0	48.3	48.8	49.1	48.5
1764	47.8	46.9	45.4	43.0	40.8	37.8	34.9	32.0	29.9	28.8	27.3	25.8	36.7
1765	25.3	25.2	24.6	23.6	22.5	21.4	20.4	19.3	19.1	19.0	18.6	18.1	21.4
1766	16.4	14.4	12.8	12.0	11.2	11.2	12.1	13.5	14.5	15.9	17.2	18.6	14.2
1767	20.6	22.9	26.0	29.3	32.9	36.4	38.9	41.5	43.1	43.7	46.1	49.9	35.9
1768	53.0	55.4	57.8	60.6	63.5	67.4	70.7	71.5	72.1	75.1	77.2	77.8	66.8
1769	81.2	86.2	91.5	97.9	103.7	106.1	107.3	111.9	115.8	114.5	112.5	111.9	103.4
1770	111.1	110.9	109.3	105.2	102.3	101.2	98.0	91.1	85.7	84.9	88.9	93.9	98.5
1771	93.6	89.1	86.1	85.4	83.5	81.9	84.3	88.9	90.1	90.5	86.9	79.5	86.7
1772	77.3	77.6	75.4	72.8	70.7	67.8	64.6	60.1	58.3	56.7	54.3	53.3	65.7
1773	50.0	46.1	43.5	40.4	37.4	35.6	34.5	35.6	37.3	38.0	38.9	39.3	39.7
1774	38.9	38.2	37.1	35.6	34.2	31.9	28.9	24.4	19.8	16.6	13.3	10.6	27.5
1775	9.3	8.6	8.5	7.9	7.5	7.2	7.7	8.9	9.2	9.4	10.2	10.7	8.8
1776	11.0	11.7	12.9	14.5	16.3	18.5	20.8	22.8	25.2	29.6	35.6	41.0	21.7

TABLE 2 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1777	47.5	55.1	62.9	70.3	78.1	87.6	98.0	106.6	113.6	119.6	128.2	138.6	92.2
1778	144.8	148.4	151.9	156.3	158.5	156.5	151.8	151.5	153.2	152.5	148.4	141.9	151.3
1779	139.0	137.5	133.8	129.9	127.0	125.7	124.1	119.4	115.7	112.8	109.4	106.9	123.4
1780	103.5	100.0	98.2	95.5	91.3	86.9	86.0	86.2	83.4	80.4	79.2	79.5	89.2
1781	79.4	78.0	75.4	71.5	69.8	69.1	66.2	62.8	60.6	58.8	55.6	51.0	66.5
1782	47.0	44.5	42.9	42.0	40.4	38.7	37.4	36.3	36.0	35.0	33.2	31.3	38.7
1783	30.6	29.4	27.7	26.4	25.1	23.6	22.2	20.3	18.3	16.9	15.5	14.1	22.5
1784	12.3	10.8	10.0	9.7	9.8	10.0	9.9	9.6	9.5	9.7	10.5	11.9	10.3
1785	13.9	15.5	16.9	19.4	22.0	23.5	25.4	28.3	31.6	36.1	42.0	46.3	26.7
1786	49.6	54.5	60.7	66.7	72.6	79.3	86.9	93.4	97.5	100.9	104.4	107.9	81.2
1787	111.4	115.3	119.2	123.0	125.9	129.5	132.2	133.3	136.6	138.0	136.4	137.8	128.2
1788	140.7	141.2	140.4	139.1	136.6	132.8	129.9	128.7	127.6	127.3	128.3	127.3	133.3
1789	124.9	122.5	119.9	116.5	116.0	117.9	117.7	117.3	116.4	114.2	111.7	109.2	117.0
1790	106.0	103.4	101.2	99.6	97.2	92.5	88.6	84.6	81.0	79.4	77.8	75.9	90.6
1791	74.9	73.1	70.8	69.4	67.9	66.9	66.0	65.4	65.1	64.5	64.0	63.4	67.6
1792	62.2	61.9	62.2	61.8	61.3	60.5	60.0	59.5	58.8	57.6	56.2	55.4	59.8
1793	55.1	54.0	51.3	49.3	48.3	47.3	46.4	45.5	44.3	42.6	41.7	41.4	47.3
1794	40.7	40.7	40.7	39.3	39.6	40.8	40.0	38.9	37.6	36.2	34.7	32.7	38.5
1795	30.5	28.7	28.2	28.0	25.8	22.7	21.3	20.6	20.1	20.8	20.9	20.1	24.0
1796	20.2	19.8	19.0	18.8	17.8	16.6	15.7	14.6	13.3	11.6	9.9	9.5	15.6
1797	8.8	8.0	7.7	7.0	6.7	6.5	5.9	5.4	5.7	5.9	5.5	4.7	6.5
1798	4.1	3.8	3.5	3.2	3.2	3.8	4.1	4.4	5.1	5.8	6.5	7.3	4.6
1799	7.8	7.8	7.5	7.6	7.3	6.8	7.0	7.1	6.6	5.9	5.4	5.9	6.9
1800	7.2	8.8	10.1	10.9	11.5	13.2	15.3	17.0	18.5	20.4	22.8	24.3	15.0
1801	25.2	26.6	28.3	30.0	32.1	33.7	34.9	36.5	37.7	38.6	39.6	40.7	33.7
1802	41.8	42.8	44.1	45.1	45.1	45.0	45.1	45.4	45.1	43.9	43.2	42.8	44.1
1803	42.4	41.7	40.8	41.2	42.5	43.1	42.9	42.6	43.2	45.1	45.7	45.2	43.0
1804	44.3	44.0	44.6	45.3	46.1	47.0	48.1	48.6	48.6	48.2	47.9	48.3	46.8
1805	48.9	49.2	48.8	47.1	44.9	43.1	41.3	39.8	38.4	37.2	36.3	35.2	42.5
1806	34.2	33.2	31.7	30.7	30.0	28.7	27.0	25.1	23.0	22.3	21.5	20.2	27.3
1807	18.9	17.6	16.3	14.7	13.0	11.1	9.6	8.7	8.0	7.1	6.8	7.0	11.6

TABLE 2 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1808	6.8	6.4	6.5	6.6	6.8	7.6	8.4	8.9	9.2	8.8	7.9	7.2	7.6
1809	6.7	6.1	5.3	4.6	4.0	3.0	2.2	1.6	1.1	1.0	0.8	0.4	3.1
1810	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1811	0.3	0.6	0.7	1.0	1.3	1.4	1.9	2.4	2.5	2.6	2.6	2.7	1.7
1812	2.5	2.9	3.7	3.7	3.9	4.6	4.5	4.4	4.8	5.5	6.4	7.0	4.5
1813	8.1	8.6	8.7	10.1	11.5	12.0	13.1	14.1	14.3	14.8	15.1	15.3	12.1
1814	15.4	15.2	14.6	14.0	13.5	13.7	13.8	14.5	16.2	17.4	17.9	19.8	15.5
1815	22.2	24.8	27.6	29.2	30.7	33.5	35.7	37.5	41.0	44.1	46.7	47.6	35.1
1816	47.3	43.4	46.1	47.7	48.7	47.3	46.2	46.2	46.7	46.3	44.0	42.8	46.1
1817	43.2	44.5	45.0	43.2	41.6	41.1	41.0	39.5	35.0	32.4	34.1	35.2	39.6
1818	34.2	32.7	31.7	31.5	31.0	30.2	30.0	29.8	28.8	27.3	25.3	23.9	29.7
1819	24.0	23.9	23.2	22.5	23.0	23.7	23.4	23.1	23.4	23.4	23.7	23.1	23.4
1820	21.7	21.2	20.8	19.6	18.1	16.5	15.8	14.9	14.1	13.7	12.1	10.6	16.8
1821	9.5	7.8	6.9	7.3	7.5	7.0	5.7	4.7	5.0	5.6	5.7	5.9	6.6
1822	6.3	6.4	6.1	5.1	4.2	4.0	4.0	4.0	3.3	2.1	1.4	1.2	4.0
1823	0.6	0.2	0.1	0.1	0.1	0.9	2.7	4.0	4.5	5.3	6.2	6.3	2.6
1824	6.3	6.3	7.2	9.1	10.2	9.4	7.9	7.4	8.5	8.8	8.6	9.8	8.3
1825	11.7	14.0	14.8	14.2	14.3	15.7	17.1	17.7	18.4	19.9	21.4	23.0	16.9
1826	24.9	26.3	27.1	28.7	31.3	34.4	37.0	38.9	41.0	42.8	44.7	46.5	35.3
1827	46.9	47.1	49.0	50.5	51.2	50.6	50.5	51.9	52.9	53.9	55.9	59.0	51.6
1828	61.2	62.6	63.6	63.5	63.8	64.2	63.8	62.8	62.4	64.1	64.7	62.8	63.3
1829	63.3	64.9	65.1	65.3	65.8	66.7	67.4	68.7	70.2	71.2	71.7	71.3	67.6
1830	68.9	65.8	65.1	66.6	68.3	69.9	70.8	69.7	69.1	67.3	63.9	61.4	67.2
1831	60.2	60.4	59.6	57.0	53.8	50.0	47.1	46.7	45.3	42.5	41.5	41.4	50.5
1832	39.8	36.6	33.4	31.1	28.9	27.6	26.7	24.2	20.7	17.9	15.7	13.5	26.3
1833	12.1	11.7	11.7	11.3	10.3	9.3	8.3	8.1	7.9	7.5	7.3	7.4	9.4
1834	7.8	7.8	7.7	8.4	10.2	12.2	13.4	13.7	14.7	17.8	21.8	24.3	13.3
1835	27.5	31.9	37.9	44.6	50.4	55.1	60.2	67.1	73.8	80.5	86.7	93.3	59.1
1836	99.5	103.9	105.7	107.2	109.9	116.1	125.6	132.6	136.9	138.2	138.0	139.4	121.1
1837	142.7	145.8	146.9	146.4	145.2	141.5	136.5	130.9	127.4	127.2	127.8	126.2	137.0
1838	121.3	116.7	113.5	111.2	108.6	105.2	101.6	100.8	98.9	93.6	87.4	82.2	103.4

TABLE 2 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1839	79.6	80.8	85.4	87.9	87.5	86.5	84.7	83.0	81.5	80.7	81.5	81.9	83.4
1840	80.7	76.6	71.1	66.9	64.6	63.6	60 8	56.0	52.5	50.5	49.4	49.7	61.9
1841	48.7	46.7	44.3	41.8	39.5	37.4	36.7	36.2	35.5	34.5	32.1	28.9	38.5
1842	26.6	25.4	24.1	23.8	25.1	25.1	23.9	22.8	21.5	20.2	19.3	18.7	23.0
1843	18.1	17.4	16 2	14.2	12.0	10.9	10.5	10.8	11.5	12.2	12.3	11.7	13.2
1844	11.9	12.9	13.5	14.3	14.6	14.6	15.7	17.6	20.0	22.7	25.7	28.4	17.7
1845	29.9	30.7	31.9	33.7	35.7	38.5	40.6	41.5	42.6	44.0	45.0	46.9	38.4
1846	49.0	50.6	54.8	58.6	60.1	61.3	62.2	63.2	63.9	63.8	63.4	64.9	59.7
1847	66.0	69.8	75.6	83.1	91.5	96.6	102.5	109.3	113.0	116.6	120.3	123.0	97.3
1848	128.3	131.6	128.7	124.2	121.1	122.2	124.2	124.9	125.3	124.6	123.5	120.8	125.0
1849	116.5	110.9	107.7	104.9	101.7	98.5	92.6	87.5	85.2	82.2	79.0	77.7	95.4
1850	75.6	74.0	73.7	73.4	71.5	68.1	66.4	67.0	66.9	66.7	67.2	67.0	69.8
1851	66.6	66.3	65.4	64.2	63.7	64.0	64.2	62.3	60.6	60.8	60.9	59.9	63.2
1852	59.5	59.0	57.0	55.9	56.2	55.3	53.1	50.9	48.9	47.2	45.6	44.5	52.8
1853	44.3	45.0	45.2	44.0	41.9	39.9	38.0	35.9	34.3	32.7	31.3	30.1	38.6
1854	28.2	25.6	23.7	22.0	20.8	20.7	20.4	20.0	19.5	18.4	16.9	15.6	21.0
1855	14.2	12.9	11.4	10.4	9.2	7.5	6.2	5.4	4.5	3.8	3.6	3.2	7.7
1856	3.3	3.6	3.9	3.9	3.8	4.1	4.9	5.5	5.8	6.2	7.6	9.3	5.2
1857	10.5	11.7	13.7	16.8	19.3	21.5	23.8	26.0	29.4	32.7	34.3	36.0	23.0
1858	38.6	41.7	44.8	48.5	51.5	53.6	56.7	60.7	64.3	67.6	71.7	75.5	56.3
1859	78.9	82.6	85.9	87.9	90.8	93.2	93.7	93.7	94.0	93.8	93.9	95.4	90.3
1860	97.2	97.9	97.0	95.4	94.4	95.1	94.9	93.7	93.3	94.5	93.6	90.6	94.8
1861	88.1	85.8	84.5	83.1	80.3	77.8	77.2	76.7	73.7	69.5	67.9	68.1	77.7
1862	67.7	66.7	65.3	63.7	62.5	60.8	58.5	57.6	58.2	58.6	57.6	55.4	61.1
1863	51.9	49.6	47.1	45.2	44.5	44.0	44.4	44.4	44.0	43.8	43.0	43.2	45.4
1864	44.8	46.0	46.6	46.6	47.2	47.5	46.6	45.9	44.4	43.1	42.5	41.3	45.2
1865	39.1	37.2	36.2	35.2	33.2	31.1	29.8	29.0	28.4	27.2	25.9	24.2	31.4
1866	22.8	21.0	19.4	18.7	17.9	16.8	15.0	12.1	9.9	8.7	7.8	6.7	14.7
1867	5.9	5.4	5.2	5.3	5.3	6.3	7.9	9.2	10.5	12.6	14.9	17.1	8.8
1868	19.3	21.5	24.2	27.6	31.7	35.5	39.2	42.9	45.8	47.1	50.5	56.9	36.9
1869	61.4	64.6	68.0	69.4	70.1	72.4	74.6	77.6	84.3	93.8	101.7	105.8	78.6

TABLE 2 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1870	110.0	116.2	121.6	127.5	134.0	138.0	139.6	140.5	140.2	139.6	138.5	135.4	131.8
1871	132.3	129.3	125.1	120.4	116.3	112.9	110.8	110.3	107.8	103.0	98.9	98.0	113.8
1872	98.9	98.3	99.0	101.0	101.9	101.9	102.0	101.7	101.6	100.9	97.4	92.2	99.7
1873	87.8	85.2	81.4	76.2	71.5	67.7	65.2	62.4	58.4	54.4	52.4	52.0	67.9
1874	51.8	51.5	50.4	49.1	47.4	45.5	42.7	39.1	36.8	36.1	34.6	32.7	43.1
1875	29.8	25.5	22.5	20.5	19.2	17.9	17.1	16.8	16.3	15.1	13.7	12.5	18.9
1876	11.7	11.6	11.7	12.0	11.8	11.4	11.7	11.9	10.8	10.6	11.8	13.0	11.7
1877	13.1	12.6	12.7	12.7	12.6	12.5	11.4	10.4	10.1	9.3	8.0	7.1	11.0
1878	6.6	6.0	5.3	4.6	4.0	3.5	3.3	3.9	2.4	2.3	2.4	2.2	3.9
1879	2.5	3.2	3.7	4.2	5.0	5.7	6.9	9.0	10.9	12.3	13.7	15.8	7.7
1880	17.7	19.8	23.9	27.6	29.7	31.3	32.8	34.4	36.8	39.5	41.6	43.6	31.6
1881	47.0	49.7	49.6	49.9	51.8	53.5	54.6	55.6	57.0	59.5	62.2	62.4	54.4
1882	60.4	58.4	57.9	57.8	58.9	59.9	60.3	60.0	58.1	56.5	54.6	54.5	58.1
1883	57.3	59.0	59.0	59.8	60.9	62.3	65.0	67.9	71.4	73.0	74.2	74.6	65.4
1884	72.4	71.7	72.4	71.3	67.8	64.6	61.4	58.8	56.6	54.2	53.6	55.2	63.3
1885	57.1	57.4	56.2	54.9	54.4	53.2	51.6	49.2	47.6	47.4	45.2	41.1	51.3
1886	37.2	34.3	32.2	30.2	27.5	25.8	24.6	23.2	20.5	16.7	14.7	13.8	25.1
1887	13.1	13.0	12.6	11.9	12.1	12.7	13.2	13.0	12.9	13.0	12.4	11.5	12.6
1888	10.3	8.6	7.9	7.8	7.8	7.3	6.3	5.8	5.8	5.8	5.6	5.3	7.0
1889	5.6	6.6	7.2	7.1	6.7	6.3	6.5	6.3	5.9	5.7	5.7	5.6	6.3
1890	5.5	5.0	5.0	5.8	6.6	7.0	7.4	8.6	9.8	10.8	13.1	16.5	8.4
1891	20.5	23.5	26.0	29.2	32.2	34.6	37.9	42.5	46.3	50.0	53.7	56.5	37.7
1892	58.4	62.0	65.2	66.4	68.1	71.0	73.2	73.4	73.9	75.3	76.3	77.0	70.0
1893	78.0	79.7	81.5	82.5	83.3	84.3	85.3	86.1	86.0	85.2	85.6	86.7	83.7
1894	87.9	86.2	83.2	82.5	81.6	79.4	77.2	75.6	75.3	75.4	73.8	71.3	79.1
1895	67.7	65.2	64.8	64.2	63.5	63.5	62.5	60.7	59.9	58.2	55.1	52.5	61.5
1896	51.5	49.6	48.0	46.5	44.5	43.0	42.3	41.6	39.5	38.0	37.1	35.2	43.1
1897	32.9	32.0	31.2	30.1	28.3	26.6	25.8	25.7	26.3	26.0	25.6	26.3	28.1
1898	26.0	25.6	25.4	25.7	27.5	27.6	26.3	24.7	22.7	21.9	21.1	20.3	24.6
1899	20.4	19.4	17.1	15.1	13.2	12.2	11.7	11.5	11.2	10.9	11.3	11.3	13.8
1900	10.7	10.5	10.6	10.6	10.4	9.9	9.1	8.2	7.6	6.8	5.9	5.4	8.8

TABLE 2 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1901	4.8	4.4	3.9	3.2	2.8	2.8	3.0	3.1	3.3	3.6	3.3	2.8	3.4
1902	2.6	2.7	3.1	3.9	4.7	5.0	5.2	6.0	6.8	7.9	9.5	10.6	5.7
1903	12.3	14.6	15.8	16.9	19.3	22.5	25.4	26.6	27.9	29.6	31.4	33.5	23.0
1904	35.5	37.7	39.7	41.1	41.5	41.6	42.9	46.4	49.8	50.5	50.7	51.3	44.1
1905	52.5	53.5	54.6	56.6	60.5	63.4	63.1	60.4	58.5	59.5	60.6	61.6	58.7
1906	63.4	64.2	63.8	61.3	55.9	53.5	55.1	59.6	62.7	62.4	61.7	60.1	60.3
1907	56.9	55.0	56.4	59.6	62.6	62.8	60.5	55.9	51.4	50.3	50.4	50.6	56.0
1908	50.5	51.6	53.2	51.9	49.9	48.9	49.3	50.5	52.6	53.1	51.9	50.6	51.2
1909	49.4	46.4	41.6	40.7	42.2	43.3	42.6	40.7	38.2	35.4	33.8	32.8	40.6
1910	31.5	30.1	29.1	27.7	24.7	20.6	17.6	15.7	14.2	14.0	13.8	12.8	21.0
1911	12.0	11.2	10.0	7.6	6.0	5.9	5.6	5.1	4.6	4.0	3.3	3.2	6.5
1912	3.2	3.0	3.1	3.4	3.4	3.4	3.7	3.9	3.8	3.5	3.2	2.8	3.4
1913	2.6	2.5	2.2	1.8	1.7	1.6	1.5	1.5	1.6	2.4	3.3	4.0	2.2
1914	4.6	5.1	5.8	6.5	7.4	8.8	10.4	12.9	16.1	18.6	20.7	24.3	11.8
1915	29.4	34.8	38.9	42.3	45.3	46.9	48.3	49.8	51.5	53.9	56.9	58.6	46.4
1916	57.8	55.6	54.0	53.7	54.6	56.3	58.3	60.2	62.1	63.3	65.1	68.7	59.1
1917	73.4	81.2	89.7	94.1	96.3	100.7	104.8	105.4	104.2	103.5	102.2	98.3	96.2
1918	95.5	92.8	88.5	87.0	87.0	83.5	78.6	77.2	77.5	76.1	75.4	78.0	83.1
1919	78.4	75.2	72.8	70.4	67.4	6 .6	63.7	62.8	61.9	60.5	56.7	51.4	65.5
1920	46.8	43.2	40.3	39.4	38.7	37.9	36.8	34.9	32.1	31.0	31.3	30.6	36.9
1921	31.0	31.7	31.1	29.0	27.3	26.5	25.3	24.4	25.5	25.8	24.3	22.5	27.0
1922	20.1	18.1	16.9	15.8	14.9	14.4	13.9	12.6	9.4	7.1	6.7	6.6	13.0
1923	6.4	5.9	6.0	6.6	6.9	6.4	5.6	5.6	5.7	5.8	6.8	8.1	6.3
1924	9.8	11.6	12.9	14.0	15.1	16.1	16.9	17.9	19.3	20.9	22.6	24.5	16.8
1925	25.9	27.1	29.3	32.6	35.9	40.9	47.2	51.8	55.6	57.7	58.9	60.9	43.7
1926	62.6	64.1	65.1	65.2	65.4	64.7	64.3	65.7	66.9	69.5	72.4	72.4	66.5
1927	72.0	71.8	71.7	71.7	71.6	70.5	69.1	68.4	68.3	68.4	67.7	69.0	70.0
1928	72.1	75.1	77.3	78.1	77.3	77.2	77.1	76.1	74.2	71.6	69.2	67.7	74.5
1929	66.2	64.3	61.3	58.6	59.6	63.0	64.8	64.0	62.8	61.1	60.6	57.5	62.0
1930	53.6	49.8	48.0	47.1	44.2	39.0	33.5	31.2	30.7	30.2	29.4	28.3	38.8

TABLE 2 (continued)

Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Average
1931	27.6	26.9	25.9	24.2	22.6	21.6	21.1	19.7	17.8	16.3	14.8	14.8	21.1
1932	14.8	14.2	13.3	12.6	12.2	11.4	11.2	11.7	12.0	11.7	10.7	9.4	12.1
1933	8.4	7.9	7.7	7.5	6.9	6.2	5.4	4.3	3.4	3.6	4.6	5.4	5.9
1934	5.7	6.3	6.6	6.7	7.2	8.1	9.4	10.6	11.9	12.7	13.0	15.0	9.4
1935	17.6	19.6	22.0	25.6	29.9	34.2	37.9	42.0	46.5	51.3	55.0	57.2	36.6
1936	59.0	62.2	65.9	68.8	72.5	77.2	82.6	87.8	90.3	92.1	96.1	101.2	79.6
1937	107.6	113.5	116.7	119.2	119.0	115.8	113.0	111.2	110.9	110.6	110.8	109.8	113.2
1938	109.3	109.2	107.9	106.3	107.1	109.4	108.8	106.3	103.6	103.0	103.0	102.8	106.4
1939	101.1	96.9	97.4	97.9	95.2	90.9	87.6	85.5	85.5	84.3	79.6	76.3	89.8
1940	74.2	73.0	71.1	67.8	66.0	66.7	67.6	66.8	64.6	61.9	59.7	57.6	66.4
1941	56.6	54.7	52.8	52.4	51.2	49.0	47.0	47.0	47.6	49.1	50.2	47.8	50.5
1942	43.7	41.1	36.5	33.3	31.8	31.0	29.6	27.7	25.6	23.0	21.1	20.5	30.4
1943	20.1	19.9	19.6	18.8	17.5	16.5	16.0	14.4	12.6	10.8	9.2	8.6	15.3
1944	8.2	7.7	7.8	8.4	8.8	9.2	10.2	11.3	12.3	14.0	16.5	19.0	11.1
1945	21.9	23.8	25.1	28.1	31.7	33.1	34.3	38.6	43.9	48.1	52.1	56.0	36.4
1946	60.6	67.0	72.9	76.8	81.4	88.6	95.3	100.2	104.3	109.6	117.6	126.2	91.7
1947	131.7	136.8	143.4	149.0	151.8	151.7	151.2	148.9	145.5	145.7	146.2	145.3	145.6
1948	144.8	142.8	140.5	138.2	135.8	135.3	136.6	141.1	147.7	148.5	143.9	139.2	141.2
1949	136.6	134.5	133.2	133.0	134.8	136.0	134.4	130.0	124.4	121.0	119.6	118.0	129.6
1950	115.0	111.9	106.4	99.5	92.9	86.6	82.2	79.0	75.3	72.2	71.4	72.2	88.7
1951	71.7	69.5	69.8	70.7	70.2	69.8	68.6	66.3	63.3	59.2	53.0	46.8	64.9
1952	43.2	42.0	39.5	36.0	33.6	31.9	30.8	29.4	28.2	27.6	27.1	26.0	33.0
1953	24.1	21.6	19.9	18.9	17.4	15.2	12.8	11.6	11.4	10.4	8.8	7.4	15.0
1954	6.4	5.6	4.2	3.4	3.7	4.2	5.4	7.2	7.8	7.9	9.5	12.0	6.4
1955	14.2	16.4	19.5	23.4	28.8	35.1	40.1	46.5	55.5	64.4	73.0	81.0	41.5
1956	88.8	98.5	109.3	118.7	127.4	136.9	145.5	149.6	151.5	155.8	159.6	164.3	133.8
1957	170.2	172.2	174.3	181.0	185.5	187.9	191.4	194.4	197.3	199.5	200.8	200.1	187.9
1958	199.0	200.9	201.3	196.8	191.4	186.8	185.2	184.9	183.8	182.2	180.7	180.5	189.5
1959	178.6	176.9	174.5	169.2	165.1	161.4	155.8	151.3	146.3	141.1	137.2	132.5	157.5
1960	128.9	125.0	121.6	119.6	117.0	113.9	108.4	101.9	97.2	92.6	87.2	82.9	108.0

TABLE 3

Quarterly Wolf numbers

Year	I	II	III	IV	Year	I	II	III	IV
1749	63.5	74.7	79.0	106.4	1803	43.4	35.1	42.6	51.1
1750	79.5	92.8	93.2	68.1	1804	47.2	39.9	42.0	61.1
1751	52.9	55.9	49.9	31.9	1805	52.2	39.0	41.6	36.2
1752	52.0	52.9	48.3	41.4	1806	33.8	26.6	26.8	25.3
1753	40.6	35.6	29.7	17.2	1807	11.3	15.3	10.1	3.5
1754	1.6	20.4	13.1	13.8	1808	1.5	13.1	8.8	9.2
1755	9.4	2.2	9.9	16.8	1809	5.8	4.1	0.3	0.0
1756	8.3	11.6	7.3	13.6	1810	0.0	0.0	0.0	0.0
1757	20.5	27.0	38.7	43.6	1811	0.0	0.0	3.0	2.7
1758	46.2	54.6	48.4	41.2	1812	4.6	0.8	7.1	7.3
1759	46.4	48.7	66.5	54.3	1813	4.1	11.1	14.0	19.6
1760	67.2	59.5	67.6	57.1	1814	13.3	14.8	9.6	18.0
1761	80.6	92.7	95.3	72.8	1815	25.9	32.4	38.1	45.2
1762	54.1	59.1	56.7	74.8	1816	56.3	48.9	36.6	41.5
1763	40.7	33.8	49.6	56.2	1817	63.5	29.2	43.9	27.6
1764	53.2	36.2	29.4	26.6	1818	29.0	41.3	28.5	22.8
1765	25.0	20.7	24.2	13.7	1819	19.0	24.9	24.1	27.7
1766	19.9	11.9	3.9	10.0	1820	16.8	19.8	17.2	8.9
1767	33.5	32.0	35.1	50.7	1821	10.5	4.2	3.9	7.7
1768	55.3	65.9	64.7	93.4	1822	5.7	6.9	3.3	0.1
1769	67.5	88.2	129.2	139.5	1823	0.2	0.0	0.2	6.8
1770	108.9	68.1	113.5	112.7	1824	10.8	7.4	7.3	8.7
1771	43.0	112.4	75.9	95.1	1825	14.3	11.5	24.0	18.4
1772	74.3	62.4	61.3	67.7	1826	10.9	31.2	37.0	52.7
1773	43.9	34.1	23.2	36.8	1827	46.6	53.0	48.7	50.5
1774	56.0	40.9	10.7	14.6	1828	60.7	82.7	60.4	46.2
1775	5.3	9.1	4.0	9.5	1829	54.9	78.8	74.0	60.4
1776	13.2	17.3	13.7	35.0	1830	69.6	79.5	52.2	82.6
1777	40.2	82.2	107.7	136.6	1831	63.7	42.0	46.0	38.5
1778	140.2	185.2	154.9	137.2	1832	47.2	31.6	10.3	21.0
1779	132.6	132.9	122.0	116.0	1833	12.7	5.6	8.1	7.8
1780	88.7	96.7	88.5	65.2	1834	9.0	6.0	8.1	29.9
1781	75.5	90.2	63.5	43.2	1835	17.2	46.1	73.2	90.9
1782	42.8	44.8	38.3	28.2	1836	98.1	126.3	106.5	154.8
1783	31.1	28.8	23.4	11.2	1837	166.1	135.8	131.0	120.2
1784	10.7	8.7	8.7	13.0	1838	123.5	119.6	86.9	82.7
1785	7.8	20.9	29.4	38.2	1839	95.9	56.7	116.2	74.4
1786	44.2	78.9	94.7	113.7	1840	74.8	61.2	64.2	52.6
1787	109.4	120.4	140.8	157.5	1841	27.9	55.2	35.1	29.0
1788	136.8	125.6	139.5	122.1	1842	21.4	24.1	19.2	32.1
1789	119.8	122.3	110.7	119.7	1843	8.4	13.5	8.5	12.4
1790	108.9	92.7	77.9	80.1	1844	12.6	12.2	17.3	17.9
1791	69.6	71.7	60.2	64.9	1845	37.5	45.3	30.8	46.6
1792	61.7	66.2	54.9	57.3	1846	51.2	64.7	69.5	60.6
1793	55.5	52.1	37.8	45.6	1847	64.4	68.5	118.0	143.0
1794	42.7	41.9	30.7	49.1	1848	126.6	111.0	124.0	135.6
1795	24.6	22.2	17.4	20.8	1849	128.3	88.1	77.7	89.4
1796	20.5	19.8	15.6	8.2	1850	83.3	58.6	62.3	61.9
1797	7.5	7.5	5.3	5.2	1851	81.8	60.8	53.8	61.6
1798	6.1	0.4	1.8	8.0	1852	65.7	55.7	39.7	55.7
1799	12.0	9.1	0.7	5.3	1853	40.6	40.8	43.3	31.8
1800	10.0	9.6	17.3	20.0	1854	18.7	23.8	19.0	20.8
1801	28.7	31.4	35.7	40.2	1855	13.7	6.3	1.2	5.7
1802	45.2	44.0	49.9	41.0	1856	1.9	3.8	5.0	6.5

TABLE 3 (continued)

Year	I	II	III	IV	Year	I	II	III	IV
1857	8.8	18.8	27.2	36.4	1910	26.4	14.3	17.3	16.3
1858	43.5	41.4	64.0	70.0	1911	6.7	9.2	3.8	3.0
1859	87.2	87.9	102.6	97.6	1912	1.7	4.3	4.3	4.0
1860	89.5	95.7	103.1	94.5	1913	1.9	0.3	1.0	2.5
1861	80.4	81.0	80.1	67.1	1914	2.8	11.3	8.6	15.6
1862	57.1	67.4	67.5	47.8	1915	34.7	47.7	63.6	43.5
1863	57.1	45.1	34.3	39.6	1916	55.9	71.3	44.6	56.4
1864	57.0	44.7	46.0	40.0	1917	80.5	101.2	134.6	99.3
1865	42.5	32.5	38.7	18.2	1918	77.8	72.2	96.4	75.9
1866	31.5	15.7	9.8	8.2	1919	64.7	83.7	62.8	43.2
1867	3.3	3.2	6.6	16.0	1920	58.4	29.9	27.7	35.6
1868	19.3	31.5	35.6	62.8	1921	28.8	29.4	27.5	18.8
1869	57.6	84.5	73.1	80.4	1922	31.0	8.3	7.4	10.4
1870	117.2	157.2	140.7	141.3	1923	3.1	6.1	5.7	8.1
1871	118.9	133.2	97.8	94.9	1924	2.3	18.7	24.2	21.5
1872	96.0	106.5	104.3	99.8	1925	15.6	40.7	45.5	75.5
1873	97.3	56.3	60.9	50.7	1926	68.1	58.8	58.2	70.5
1874	57.1	38.3	52.7	30.8	1927	81.4	77.2	59.0	58.5
1875	23.5	21.5	9.8	13.4	1928	80.8	83.0	90.5	56.9
1876	20.2	3.0	11.3	10.8	1929	61.1	61.0	56.8	81.0
1877	12.9	16.8	9.5	7.8	1930	49.8	34.6	26.3	31.9
1878	5.7	4.1	1.8	1.9	1931	29.2	23.7	16.5	15.5
1879	0.5	4.5	8.1	10.8	1932	11.3	17.1	6.8	9.4
1880	23.7	25.6	45.3	34.4	1933	14.9	3.8	2.7	1.3
1881	47.0	51.9	62.7	55.4	1934	5.2	12.6	7.2	9.9
1882	60.6	68.4	47.8	61.8	1935	20.8	28.4	35.4	59.6
1883	50.1	63.6	59.7	81.4	1936	71.4	66.5	71.8	109.3
1884	88.4	64.6	56.9	43.9	1937	115.0	118.8	127.8	96.0
1885	54.8	70.6	52.0	31.6	1938	101.4	108.6	123.5	104.7
1886	37.7	33.8	22.9	7.1	1939	74.1	109.5	105.3	66.1
1887	9.2	9.0	17.4	11.4	1940	64.4	66.3	79.8	60.6
1888	9.2	6.4	4.9	6.5	1941	45.5	40.7	64.3	39.4
1889	5.4	4.4	12.3	3.0	1942	47.5	32.4	18.4	24.1
1890	3.7	2.6	12.4	9.5	1943	22.9	15.9	14.2	12.3
1891	15.4	36.6	48.6	41.9	1944	5.1	2.6	12.0	18.7
1892	64.9	75.2	80.3	71.5	1945	17.6	32.9	34.5	47.4
1893	71.2	87.0	98.6	82.9	1946	70.1	78.0	105.9	115.9
1894	73.4	93.9	80.7	64.0	1947	126.3	171.7	172.0	136.0
1895	63.8	72.0	58.1	61.9	1948	96.5	177.2	147.8	126.7
1896	46.1	40.2	44.5	36.3	1949	153.0	125.0	131.6	130.9
1897	33.0	20.8	32.5	18.7	1950	102.0	101.1	75.8	56.8
1898	35.0	20.9	25.1	26.0	1951	55.2	100.7	68.5	49.9
1899	15.6	14.1	8.3	10.4	1952	28.5	29.6	40.8	26.7
1900	10.5	14.4	7.0	5.9	1953	13.5	20.7	17.1	4.1
1901	2.4	5.3	0.8	2.5	1954	3.9	0.9	4.9	7.9
1902	5.9	1.4	3.6	9.2	1955	16.3	24.0	36.7	74.9
1903	12.9	19.0	22.6	43.0	1956	105.3	121.3	157.3	182.9
1904	31.1	41.5	46.3	48.9	1957	150.9	180.2	193.7	234.7
1905	65.7	45.4	62.3	90.5	1958	189.4	180.9	197.6	173.8
1906	47.1	58.7	69.0	40.5	1959	182.1	168.0	164.8	120.1
1907	81.8	45.3	63.0	58.1	1960	118.2	117.3	127.7	86.0
1908	33.9	48.8	72.3	39.1	1961	52.3	63.3	63.2	36.7
1909	56.5	30.3	32.6	56.1					

REFERENCES*

ALFVÉN, H. — 1952. Kosmicheskaya elektrodinamika (Cosmical Electrodynamics), p. 147. — Izdatel'stvo inostrannoi literatury. Moskva [Translated from English].

BEZRUKOVA, A. Ya.
1951. Byulleten' KISO, No. 7 (21).
1957. Soln. dannye, No. 2.
1958. Izv. GAO, No. 159.
1959a. Soln. dannye, No. 6.
1959b. Soln. dannye, No. 11.

BOGORODSKII, A. F. and E. N. ZEMANEK. — 1950. Publikatsii Kievskoi astronomicheskoi observatorii, No. 3.

BONOV, A. D.
1957. Soln. dannye, No. 3.
1961. Soln. dannye, No. 3.

VITINSKII, Yu. I.
1956a. Soln. dannye, No. 1.
1956b. Soln. dannye, No. 4 : 148.
1956c. Soln. dannye, No. 4 : 150.
1957. Soln. dannye, No. 4.
1958a. Soln. dannye, No. 3.
1958b. Soln. dannye, No. 6.
1960a. Izv. GAO, No. 163 (88).
1960b. Soln. dannye, No. 8.
1960c. Soln. dannye, No. 9.
1960d. Soln. dannye, No. 10.
1960e. Soln. dannye, No. 11.
1961a. Soln. dannye, No. 1.
1961b. Izv. GAO, No. 167.
1961c. AZh, Vol. 38, No. 4.
1961d. Soln. dannye, No. 11.

VITINSKII, Yu. I. and B. M. RUBASHEV. — 1957. Soln. dannye, No. 2.

VITINSKII, Yu. I. and R. N. IKHSANOV. — 1960. Soln. dannye, No. 1.

VSEKHSVYATSKII, S. K. — 1950. Publikatsii Kievskoi astronomicheskoi observatorii, No. 3.

GIMMEL'FARB, B. N. — 1950. Byulleten' KISO, No. 5—6 (19—20).

GNEVYSHEV, M. N.
1938. Tsirkulyary GAO, No. 24.
1944. Kosmicheskie dannye, No. 1.

* [Russian-language references are given first, in Cyrillic-alphabet order; they are followed by the non-Russian references, in Latin-alphabet order. A brief list of the Russian abbreviations used here follows the references.]

GNEVYSHEV, M. N. and A. I. OL'. — 1948. AZh, Vol. 38, No. 1.

GNEVYSHEV, M. N. and R. S. GNEVYSHEVA. — 1949. Byulleten' KISO, No. 1(15).

IVANOV, N. I.
1933. Tsirkulyary GAO, No. 22.
1935. Tsirkulyary GAO, No. 39.
1936. Tsirkulyary GAO, No. 52.

IKHSANOVA, V. N. — 1960. Izv. GAO, No. 164.

KOZIK, S. M.
1946. AZh, Vol. 23, No. 4.
1949. AZh, Vol. 26, No. 1.

MAKSIMOV, I. V. — 1953. Doklady Akademii nauk, Vol. 92, No. 6.

MOLCHANOV, A. P. — 1959. Soln. dannye, No. 4.

OL', A. I.
1949a. Byulleten' KISO, No. 1(15).
1949b. Byulleten' KISO, No. 2(16).
1954. In: Voprosy dolgosrochnykh prognozov solnechnoi aktivnosti (Problems of Long-Range Forecasts of Solar Activity). — Izdatel'stvo Akademii nauk SSSR.
1960. AZh, Vol. 37, No. 2.
1961. Soln. dannye, No. 1.

ROMANOVSKII, V. I. —1947. Primeneniya matematicheskoi statistiki v opytnom dele (Applications of Mathematical Statistics in Practice). —GITTL, Moskva.

RUBASHEV, B. M.
1949. Byulleten' KISO, No. 2(16), p. 31.
1954. In: Voprosy dolgosrochnykh prognozov solnechnoi aktivnosti (Problems of Long-Range Forecasts of Solar Activity). — Izdatel'stvo Akademii nauk SSSR.
1958. Izv. GAO, No. 162.

CHISTYAKOV, V. F. — 1959. Soln. dannye, No. 2.

EIGENSON, M. S.
1940. Tsirkulyary GAO, No. 30.
1947. Priroda, No. 6(3).
1955. Tsirkulyary L'vovskoi astronomicheskoi observatorii, No. 9(25).

EIGENSON, M. S., M. N. GNEVYSHEV, A. I. OL', and B. M. RUBASHEV. — 1948. *Solnechnaya aktivnost' i ee zemnye proyavleniya* (Solar Activity and Its Geophysical Effects). — GITTL, Moskva-Leningrad.

EIGENSON, M. S. and I. A. PROKOF'EVA. — 1950. Byulleten' KISO, No. 5-6 (19-20).

EIGENSON, M. S. and T. L. MANDRYKINA. — 1954. Astronomicheskii sbornik L'vovskogo universiteta, Vol. 2, 62.

ALTER, D. — 1928. Monthly Weath. Rev., Oct.

ANDERSON, C. N.
1939. Bell Syst. Tech. J., Vol. 18, 292.
1954. J. Geoph. Res., 59, No. 4.

BALLI, E. — 1955. Ann. Astroph., Vol. 18, No. 2.

BAUR, F. — 1961. Zs. Astroph., Vol. 52, No. 1.

BECKER, U.
1954. Zs. Astroph., Vol. 35, No. 3.
1955. Zs. Astroph., Vol. 37, No. 1.
1959. Zs. Astroph., Vol. 48, No. 2.

BECKER, U. and K. O. KIEPENHEUER. — 1953. Zs. Astroph., Vol. 33, No. 2.

BELL, B. — 1960. Smithson. Contr. Astroph., Vol. 5, No. 3.

BJERKNES, V. — 1926. Ap. J., Vol. 64, No. 1.

BRUNNER-HAGGER, W. and A. LIEPERT. — 1941. Astr. Mitt. Zürich, Vol. 14, No. 140.

CARRINGTON, R. C. — 1858. M. N., Vol. 19.

CHADWICK, W. B. — 1959. Nature, Vol. 184, No. 4701.

CHERNOSKY, E. J. — 1954. PASP, Vol. 66, No. 392.

CHVOJKOVA, E. — 1952. BAC, Vol. 3, No. 2.

CIMINO, M. — 1944. Comm. Pont. Acad. Sci., Vol. 8, 485.

CLAYTON, H. H. — 1939. The Sunspot Period. Washington.

COOK, A. F. — 1949. J. Geoph. Res., Vol. 54, No. 4.

DIZER, M. — 1957. Ann. Astroph., Vol. 19, No. 4.

DJURCOVIČ, P.
1953. Publ. Obs. Astr. Beograd, Vol. 18, No. 1—2.
1956. Soln. dannye, No. 6 [In Russian].

DYSON, F. — 1925. M. N., Vol. 85, No. 6.

GLEISSBERG, W.
1939. Astr. Nachr., Vol. 268, No. 6414.
1940. Obs., Vol. 63, No. 795.
1942. Ap., J., Vol. 96, No. 2.
1944a. Obs., Vol. 65, No. 822.
1944b. Terr. Magn., Vol. 49, No. 4.
1945. Obs., Vol. 66, No. 827.
1946. Publ. Istanbul Obs., No. 30.

1949. Ap. J., Vol. 110, No. 1.
1951a. Publ. Istanbul Obs., No. 41.
1951b. Zs. Astroph., Vol. 29, No. 1.
1952. Die Häufigkeit der Sonnenflecken. Berlin.
1953. Zs. Astroph, Vol. 31, No. 1.
1955. Zs. Astroph., Vol. 37, No. 2. .
1958. Zs. Astroph., Vol. 46, No. 4.
1960. Zs. Astroph., Vol. 49, No. 1.

GRANGER, C. W. J. — 1957, Ap. J., Vol. 126, No. 1.

HALE, G. E. — 1913. Ap. J., Vol. 38, No. 1.

HALE, G. E. and S. B. NICHOLSON. —1938. Magnetic Observations of Sunspots, I, II, Washington.

HALM, J. — 1901. Astr. Nachr., Vol. 156, No. 3723-3724.

HERRINK, P.
1958. Bull. Ac. Roy. Sci. Col., Nouv. Ser., 4(6), 1274.
1959. Nature, Vol. 184, No. 4679.

KIMURA, H. — 1913. M. N., Vol. 73, No. 7.

KOPECKÝ, M.
1950a. BAC, Vol. 2, No. 1.
1950b. BAC, Vol. 2, No. 2.
1958. BAC, Vol. 9, No. 1.
1959. Publ. Astr. Inst. Czecosl. Ac. Sci., No. 42.
1960. BAC, Vol. 11, No. 4.
1962. BAC, Vol. 13, No. 2.

KORTEWEG.—1883. Sitzungsb. Wiener Akad., Vol. 88, Abt. II.

LOCKYER, W. J. S. — 1904. M. N., Vol. 64, No. 1.

LOEWE, F. and U. RADOK. — 1959. Gerl. Beitr. Geoph., Vol. 68, No. 6.

LOSH, H. M. — 1938. Publ. Obs. Michigan, Vol. 7, No. 5.

LUDENDORFF, H. — 1931. Zs. Astroph., Vol. 2, No. 5.

MACNISH, A. G. and J. V. LINCOLN. — 1949. Frans. Amer. Geoph. Un., Vol. 30, No. 5.

MAUNDER, E. W.
1890. M. N., Vol. 50, No. 6.
1904. M. N., Vol. 64, No. 8.
1917. M. N., Vol. 77, No. 8.

MAYOT, M.
1947. Ann. Astroph., Vol. 10, No. 3.
1951. Ann. Astroph., Vol. 14, No. 2.

MICHELSON, A. A. — 1913. Ap. J., Vol. 38, No. 3.

MINNIS, C. M. — 1960. J. Atm. Terr. Phys., Vol. 20, No. 2/3.

NEWCOMB, S. — 1901. Ap. J., Vol. 13, No. 1.

OPPENHEIM, S. — 1928. Sitzungsb. Akad. Wiss. Wien, math-naturw., Kl. 11a, Vol. 137, No. 1—2.

SCHOVE, D. J. — 1955. J. Geoph. Res., Vol. 60, No. 2.

SCHUSTER, A. — 1906. Phil. Trans. Roy. Soc., Vol. A206, 69.

SCHWABE, S. H. — 1844. Astr. Nachr. Vol. 21, No. 495.

SPÖRER, G. — 1881. Potsdam Publ., Vol. 2.

STEWART, J. Q. and H. A. PANOFSKY. — 1938. Ap. J., Vol. 88, No. 4.

STEWART, J. Q. and F. C. EGGLESTON. — 1939. Phys. Rev., Vol. 55, 1102.

STUMPFF, K. — 1930. Cechoslov. Statistic, Vol. 65, 54.

THIELE. — 1859. Astr. Nachr., Vol. 49, No. 1193.

THÜRING, B. — 1955. Astr. Nachr., Vol. 282, No. 1.

TUOMINEN, J. — 1952. Zs. Astroph., Vol. 30, No. 4.

TURNER, H.
1913a. M. N., Vol. 73, No. 9.
1913b. M. N., Vol. 74, No. 2.

WALDMEIER. M.
1935. Astr. Mitt. Zürich, Vol. 14, No. 133.
1936. Astr. Nachr., Vol. 259, No. 6208.
1939. Astr. Mitt. Zürich, Vol. 14, No. 138.
1946. Terr. Magn., Vol. 51, No. 2.
1955. Ergebnisse und Probleme der Sonnenforschung. Leipzig.

XANTHAKIS, J.
1957. AZh, Vol. 34, No. 3 [In Russian].
1959. Ann. Astroph., Vol. 22, No. 6.

ABBREVIATIONS USED IN REFERENCE LIST

Abbreviation	Transliteration	Translation
AZh	Astronomicheskii zhurnal	Astronomical Journal
Izv. GAO	Izvestiya Glavnoi astronomicheskoi observatorii	Bulletin of the Main Astronomical Observatory
GITTL	Gosudarstvennoe izdatel'stvo tekhnicheskoi i teoreticheskoi literatury	State Technical and Theoretical Press
KISO	Komissiya po issledovaniyu solntsa	Solar Research Commission
Soln. dannye	Solnechnye dannye	Solar Data

INDEX